

A Note of Caution on Using Hotelling Models in Platform Markets

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Abstract

We study a Hotelling framework in which customers first pay a monopoly platform to enter the market before deciding between two competing services on opposite ends of a Hotelling line. This setup is common when modeling competition in Internet content provision. We find that standard taken-for-granted solution methods under full market coverage break down, and that in the unique full-coverage equilibrium, the competing service providers set substantially lower prices. Standard methods and prices are restored by giving service providers the first move.

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1 Introduction

The Hotelling framework has become a methodological mainstay in much of the platform market research. The framework has been used to model competition between rival platforms (e.g., Rochet and Tirole 2003; Armstrong 2006; Hagiu 2009), or, as has often been the case in the literature on network neutrality, competition between rival content providers (CPs) who rely on a monopolistic Internet service provider (ISP) platform.¹ In the latter characterization, which is of interest to us, researchers generally suppose that consumers must first pay a “hookup” fee to the monopoly platform, after which they select a CP whose content is consumed. This can be further segmented into models in which consumers obtain content at no additional cost beyond the hookup fee where content providers subsist on advertising or some other indirect revenue component (Choi and Kim 2010; Guo et al. 2010; Cheng et al. 2011) and those in which CPs additionally charge consumers a subscription fee (Brito et al. 2013; Baranes 2014; Fudickar 2015).

The latter situation is exemplified by, for instance, competition between streaming video service providers like Netflix or Hulu, who charge broadband Internet consumers for content. To make the equilibrium interesting, researchers generally assume a parametrization that ensures full coverage over the Hotelling line; otherwise, CPs effectively do not compete with each other (for the marginal consumer). Then, the game is typically solved as follows. The ISP is assumed to have the first move, presumably to indicate its indispensability to the market (Gee 2012 is an exception), following which CPs set prices. The game is solved by backward induction, whereby CPs’ demand functions are determined by the marginal consumer and CPs simultaneously set prices to maximize profit. Then, the ISP sets a hookup fee that leaves the marginal consumer indifferent between exiting or remaining in the market and, in the event that net neutrality is violated, some “discriminatory” charge to a CP seeking prioritization or higher quality service.

However, this simple approach ignores the ISP’s first mover advantage because the hookup fee drops out of the marginal consumer’s initial indifference equation. This leads CPs to set prices to those that would result in a standard Hotelling setup without a platform player. We show that the platform has a profitable deviation at these prices and compute the correct,

¹We note that following the maximum differentiation result of D’Aspremont et al. (1979), correcting a deficiency in Hotelling (1929), it has become conventional in these articles to suppose that rivals using the platform sit on the opposite ends of the Hotelling line.

unique, full coverage equilibrium. We find that the standard full coverage set of prices is restored by giving competing CPs the first move.

2 Model

Suppose that a monopolistic ISP serves as a platform connecting consumers to two competing CPs, A and B . CP_A is located at point 0 and CP_B is located at point 1 of a unit line with a mass 1 of consumers uniformly distributed over the line. Suppose that CPs are symmetrically differentiated with marginal cost zero. A consumer located at point $x \in [0, 1]$ on the line faces “transportation cost” tx when buying content from CP_A and $t(1-x)$ when buying from CP_B , where $t > 0$ indicates the degree of product differentiation.

Consider a game in which the ISP first charges a hookup fee H for unlimited Internet service, after which each $CP_i \in \{A, B\}$ charges all consumers that choose it a price p_i . Following Choi and Kim (2010), suppose that consumers purchase content from at most one CP. We abstract from issues related to network congestion. A consumer located at x obtains utility

$$U = \begin{cases} v - H - tx - p_A & \text{from } CP_A, \\ v - H - t(1-x) - p_B & \text{from } CP_B. \end{cases} \quad (1)$$

Assuming full market coverage, the marginal consumer x^* who is indifferent between CPs A and B satisfies:

$$v - tx^* - p_A - H = v - t(1-x^*) - p_B - H. \quad (2)$$

Solving for x^* yields:

$$x^* = \frac{p_B - p_A + t}{2t}. \quad (3)$$

Working backwards, CPs A and B set prices to maximize profits, respectively $p_A x^*$ and $p_B(1-x^*)$, to yield equilibrium prices $p_A^* = p_B^* = t$ and equally split the market (i.e., $x^* = 1/2$).² Substituting p_A^* , p_B^* , and $x^* = 1/2$ into consumers’ utility of choosing CP_i , $U_i(x, p_i, H)$, yields

²This is the standard Hotelling solution with linear transportation cost and no marginal production cost. It is readily verified that the matrix of second order price partial derivatives of profits is negative definite, so that the equilibrium is locally strictly stable.

the indifference utility:

$$U_i(x^*, p_i^*, H) = v - 3t/2 - H. \quad (4)$$

Assuming the ISP cannot discriminate across consumers when charging its hookup fee, it sets the fee to $H^* = v - 3t/2$ and extracts the indifferent consumer's entire surplus. The ISP's equilibrium profit equals $\pi_{\text{ISP}}^* = v - 3t/2$ and the CPs' profits are $\pi_A^* = \pi_B^* = t/2$.

Unfortunately, the prior, seemingly standard, Hotelling methodology (which is relied upon in some of the aforementioned papers) incorrectly ignores the ISP's first-mover advantage. The omission results from a failure to consider that CPs face a constrained optimization problem and that in the correct solution, CP prices are bounded by the constraint that the indifferent consumer's utility equals zero conditional on the equilibrium value of H .

To obtain the correct solution, we instead solve for the full market coverage equilibrium by taking the limit of the solution under partial coverage as it approaches full coverage. Consider a set of parameters that leads to partial market coverage and define x_i as the consumer who is indifferent between $\text{CP}_i \in \{A, B\}$ and leaving the market. Demand for CPs A and B can respectively be specified as:

$$x_A = \frac{v - p_A - H}{t}, \quad 1 - x_B = \frac{v - p_B - H}{t}, \quad (5)$$

where $x_A \leq x_B$. Under partial coverage, for any H , CP_i solves:

$$\max_{p_i} p_i \left(\frac{v - p_i - H}{t} \right) \text{ s.t. } x_A \leq x_B. \quad (6)$$

When the constraint in Expression (6) does not bind, simultaneous solutions to CPs' first-order conditions yield equilibrium prices $\hat{p}_A(H) = \hat{p}_B(H) = (v - H)/2$ and equilibrium quantities $\hat{x}_A(H) = 1 - \hat{x}_B(H) = (v - H)/2t$. Given $\hat{x}_A(H)$ and $\hat{x}_B(H)$, the ISP sets H to solve:

$$\max_H H \left(\frac{v - H}{t} \right) \text{ s.t. } U_i(\cdot) \geq 0. \quad (7)$$

At the CPs' equilibrium prices and quantities, $U_i(\cdot) = 0$ for all values of H , so that the constraint can be ignored. Solving the first-order condition for the equilibrium hookup fee yields $\hat{H} = v/2$, $\hat{p}_A = \hat{p}_B = v/4$, and $\hat{x}_A = 1 - \hat{x}_B = v/4t$.

If we assume that $\hat{x}_A(H) = \hat{x}_B(H) = x^* = 1/2$, which is the limiting case for the equilibrium quantities that solve Expression (6) as coverage increases from partial to full, then it must be that $H = v - t$ and $\hat{p}_A = \hat{p}_B = t/2$.

Our assumption that $\hat{x}_A < \hat{x}_B$ implies that $v < 2t$. A common approach employed when using Hotelling style models is to then assume that v is sufficiently large to entail full market coverage and to rely on Equations (2) and (3) to solve for equilibrium. As we next show, this is where CP prices are constrained by the indifferent consumer's utility. Let us then assume that $v > 2t$ and consequently $x_A = x_B$. Then, given H , $CP_i, i \neq j \in \{A, B\}$ solves:

$$\max_{p_i} p_i \left(\frac{p_j - p_i + t}{2t} \right) \text{ s.t. } U_i(\cdot) = v - tx^* - p_A - H \geq 0. \quad (8)$$

If the constraint in Expression (8) does not bind, then as we know from Equations (2) through (4), $x_A = x_B = x^* = 1/2$, $p_A^* = p_B^* = t$, and $H^* = v - 3t/2$. However, if it can continue to sell to the entire market, the ISP would clearly be better off by setting $H = v - t$, which would violate the constraint in Expression (8). In this case, we already know that $\hat{p}_A = \hat{p}_B = t/2$ is a candidate pair of equilibrium prices and as we show in Proposition 1, CPs can do no better. Moreover, when $v > 2t$, the ISP cannot profitably deviate to an H higher than $v - t$ (a lower H is suboptimal because it does not increase the number of customers).

Proposition 1. *Suppose that after the ISP charges hookup fee H , CPs set prices for content and each consumer purchases content from a single CP. Then, if $v > 2t$, in the unique subgame perfect Nash equilibrium outcome of this game, the ISP sets equilibrium hookup fee $\hat{H} = v - t$ and CPs fully cover and equally split the market with equilibrium prices $\hat{p}_A = \hat{p}_B = t/2$.*

The proof of Proposition 1, which can be found in the Appendix, proceeds in three parts. We first show that given $H = v - t$, and rival price $t/2$, a CP cannot profitably deviate to a price other than $t/2$. Next, we show that given CPs' best response functions, the platform cannot do any better than $H = v - t$. To complete the proof, we show that the symmetric CP pricing outcome is unique.

Figure 1, which graphs inverse demand for CP_A when $H = \hat{H}$, provides intuition for Proposition 1, and correspondingly, captures why the approach in Equations 2 to 4 is flawed. As the figure shows, inverse demand has an outward kink, above which the slope equals $-t$, and below

which the slope equals $-2t$. Given \hat{H} , above the kink, prices are sufficiently high to lead to partial coverage, whereas below it, full coverage emerges in equilibrium. By ignoring the ISP's first-mover advantage, Equations 2 to 4 lead to CP prices above the kink.

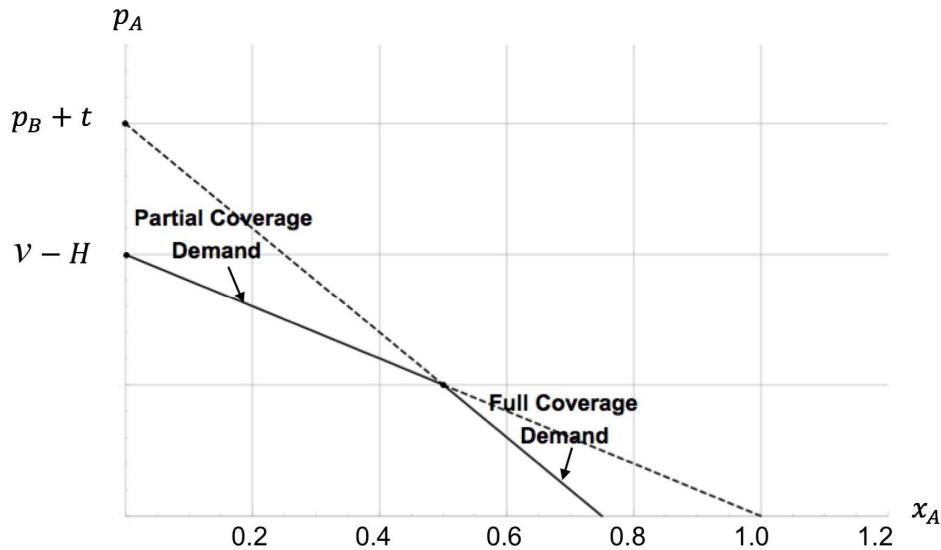


Figure 1: Best Responses and Non-existence of Equilibrium

By setting H , the ISP determines the location of the kink and the CP price above which partial coverage would result. Observe that in equilibrium, the dashed curve in Figure 1 intersects the vertical axis at $\hat{p}_B + t = 3t/2$, which is greater than $v - \hat{H} = t$. Thus, if $H = v - 3t/2$ or lower, partial coverage does not occur. However, as long as a value of H above $v - 3t/2$ still results in full coverage, the ISP will prefer to raise its hookup fee, and cause a kink to form. At the ISP's optimal hookup fee, the kink occurs at $\hat{p}_A = t/2$. Thus, CP_A faces elasticity higher than -1 above the kink and elasticity lower than $-1/2$ below it, maximizing total revenue (which, here, equals total profit) by pricing at the kink.

However, when H is too high—say some \bar{H} slightly above \hat{H} —the region immediately above the kink becomes inelastic, inducing the CPs to best respond with prices that lead to partial coverage. In contrast, because the ISP loses twice as many customers as the CPs do when the CPs price above the kink, it views demand as elastic at \bar{H} and prefers to lower its fee to \hat{H} . However, at \hat{H} , the ISP achieves full coverage and does not price any lower.

Proposition 1 states that the supposed full coverage equilibrium derived using Equations (2) through (4) incorrectly leads to an ISP hookup fee that is too low and CP prices that are too high. We next consider how altering the timing of the game can restore the outcome obtained

in these equations. That is, suppose instead that the CPs set prices ahead of the ISP hookup fee. Clearly in this case, assuming full coverage, the ISP cannot claim a first-mover advantage, and consequently, Equations (2) to (4) apply. Thus, it only remains to check whether or not the ISP has an incentive to deviate to an H that leads to partial coverage.

In order to hone in on the parameter space under which full coverage is preferred, let us again start by solving for the partial coverage equilibrium. Thus, consider the partial coverage demands given in Expression (5) and suppose that $x_A < x_B$. Working backward, for any p_A and p_B , the ISP solves:

$$\max_H H(x_A + 1 - x_B) = H \left(\frac{2v - p_A - p_B - 2H}{t} \right) \quad \text{s.t.} \quad x_A < x_B. \quad (9)$$

The ISP's first order condition yields equilibrium hookup fee $H^* = (2v - p_A - p_B)/4$ and equilibrium quantities $x_A^* = (2v - 3p_A + p_B)/(4t)$ and $1 - x_B^* = (2v - 3p_B + p_A)/(4t)$.³ Given x_A^* and x_B^* , CPs simultaneously set p_A and p_B to maximize profit. $CP_i, i \neq j \in \{A, B\}$ solves:

$$\max_{p_i} p_i \left(\frac{2v - 3p_i + p_j}{4t} \right) \quad \text{s.t.} \quad U_i(\cdot) \geq 0. \quad (10)$$

Simultaneously solving CPs' first order conditions yields equilibrium prices $p_A^* = p_B^* = 2v/5$.⁴ Accordingly, the hookup fee equals $H^* = 3v/10$ and $x_A^* = 1 - x_B^* = 3v/(10t)$. Thus, partial coverage ($x_A^* < x_B^*$) occurs if and only if $v < 5t/3$.

Because we are interested in full market coverage, let us assume that $v \geq 5t/3$. As Proposition 2 (proved in the Appendix) states, v needs to be somewhat higher than $5t/3$ in order for $p_A^* = p_B^* = t$, the equilibrium prices stemming from Equation (3) to come about. Indeed, as it turns out, the same level of v , $v \geq 2t$, that results in full coverage in a game where the ISP moves first, permits the Hotelling outcome incorrectly computed in Equations (2) to (4) to arise in equilibrium when the CPs move first instead.

Proposition 2. *Suppose that after CPs simultaneously set prices p_A and p_B , the ISP sets hookup fee H and each Internet consumer purchases content from a single CP. Then, if $v \in [5t/3, 2t]$, in the unique symmetric equilibrium, the optimal hookup fee equals $H^* = t/2$ and*

³It is easily verified that the second order condition is satisfied.

⁴Again, the matrix of second order price partial derivatives of profits is negative definite.

CPs charge prices $p_A^* = p_B^* = v - t$ and equally split the market. If $v \geq 2t$, then hookup fee $H^* = v - 3t/2$ and CP prices $p_A^* = p_B^* = t$ comprise a full coverage equilibrium outcome.

When v surpasses a certain threshold (in this case $5t/3$), a full coverage equilibrium exists. However, when CPs price at t in equilibrium, if v is not high enough, the ISP does not get sufficiently compensated under full coverage and wishes to deviate to partial coverage by setting a higher hookup fee. Once v reaches $2t$, by setting their prices to t , the CPs nevertheless leave enough consumer surplus to the ISP to keep it from wanting to deviate to a higher price.

3 Conclusion

Our work highlights how the full market coverage assumption can lead researchers to erroneously ignore an ISP's first mover advantage in a Hotelling framework with a monopolistic ISP platform. In actuality, the CP prices that prevail in this case will lead the ISP to set a hookup fee that leads to partial, not full coverage, in equilibrium. The work indicates that when choosing this admittedly convenient framework to model broadband platforms and content provision, researchers must keep in mind that CP prices are constrained by the utility of the indifferent consumer, *as well as* the ISP's price, unless the ISP has the first move.

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Appendix

Proof of Proposition 1.

Proof. As stated earlier, the proof of Proposition 1, proceeds in three parts. In Step 1, we show that given $H = v - t$, and rival price $t/2$, a CP cannot profitably deviate to a price other than $t/2$. In Step 2, we show that given CPs' best response functions, the platform cannot do any better than $H = v - t$. In Step 3, we show that there are no asymmetric CP pricing outcomes.

Step 1. Suppose that $H = v - t$. If we substitute $p_j = t/2$ into the objective function in Expression (8) and solve the maximization problem for p_i , it becomes evident that the constraint binds—the unconstrained optimum price of $p_i = 3t/4$ would leave the indifferent consumer with negative utility from consuming content from CP_i —and because the objective function is concave in p_i , $t/2$ is the best that CP_i can do under full coverage (that is, there is no profitable deviation to a lower price). Thus, if CP_i raises price in response to $p_j = t/2$, it must be that $x_A < x_B$ (because at $\hat{H} = v - t$ and $\hat{p}_i = \hat{p}_j = t/2$, the indifferent consumer gets zero utility). However, by substituting $\hat{H} = v - t$ into Expression (6), we see that the price that maximizes each CP's profit equals $t/2$ —the price that prevails under full coverage—so profits cannot be increased via partial coverage.

Step 2. Suppose that CPs price according to their best responses to \hat{H} . We show that in this case, the ISP cannot do any better than $H = v - t$. First, from the negative definiteness of the matrix of second order price partial derivatives of CP profits given in Expression (8), we know that when $v > 2t$, any hookup fee below $H = v - t$ leads to full coverage. But this means that any hookup fee below $H = v - t$ cannot be profit maximizing.

Suppose to the contrary that $H = v - t + \varepsilon$ for some $\varepsilon > 0$. From Expression (8) we know that were the CPs to then set prices to achieve full market coverage, they would be bound by the non-negative utility constraint and set prices to $\bar{p}_A(x) = t(1 - x) - \varepsilon$ and $\bar{p}_B(x) = tx - \varepsilon$. We proceed as follows: we first show that when $H = v - t + \varepsilon$, there is no set of equilibrium CP prices that would lead to full coverage; we then show that given the partial coverage equilibrium that prevails, the ISP is better off lowering its fee to $H = v - t$.

Suppose that under full coverage, $x \geq 1/2$. In this case, if CP_B sets price $\bar{p}_B(x) = tx -$

ε under full coverage, substituting $\bar{p}_B(x)$ into the objective function in Expression (8) and solving CP_A 's maximization problem for p_A , it becomes evident that the constraint binds—the unconstrained optimum price of $p_A = [t(1+x) - \varepsilon]/2$ is strictly higher than $\bar{p}_A(x)$ for $x \geq 1/2$ and would lead the indifferent consumer with negative utility from consuming content from CP_A —and because the objective function is concave in p_A , $\bar{p}_A(x)$ is the best that CP_A can do under full coverage. CP_A 's profit when CPs set prices $\bar{p}_A(x)$ and $\bar{p}_B(x)$ equals $\bar{\pi}_A(x) = [t(1-x) - \varepsilon]x$, which is strictly lower than its full coverage profit when $x = 1/2$ ($\bar{\pi}_A(1/2) = (t/2 - \varepsilon)/2$) for all $x \in (1/2, 1]$. Under partial coverage, from Expression (6), we can see that $\tilde{p}_A = (t - \varepsilon)/2$ and profits for CP_A are:⁵

$$\tilde{\pi}_A = \frac{t - \varepsilon}{2} \left[\frac{v - (t - \varepsilon)/2 - v + t - \varepsilon}{t} \right] = \frac{t}{4} - \frac{\varepsilon}{2} + \frac{\varepsilon^2}{4t} > \frac{t}{4} - \frac{\varepsilon}{2} = \bar{\pi}_A(1/2). \quad (11)$$

Thus, for any $x \in [1/2, 1]$, CP_A deviates to partial coverage. Similarly, it can be shown that for any $x \in [0, 1/2]$, CP_B would deviate to partial coverage such that $\tilde{\pi}_B = \tilde{\pi}_A$. Moreover, by checking second order conditions, it is readily verified that the symmetric partial coverage equilibrium that prevails when $H = v - t + \varepsilon$ is unique.

Expression (11) implies that when the ISP sets $H = v - t + \varepsilon$, CPs set high enough prices that $x_A < x_B$ regardless of v . Substituting $H = v - t + \varepsilon$ into the objective function in Expression (7), we see that under partial coverage, the ISP earns:

$$\tilde{\pi}_{\text{ISP}} = (v - t + \varepsilon) \left(\frac{t - \varepsilon}{t} \right). \quad (12)$$

Comparing $\tilde{\pi}_{\text{ISP}}$ with $\pi_{\text{ISP}}^* = \hat{H}$ yields:

$$\pi_{\text{ISP}}^* = v - t > (v - t + \varepsilon) \left(\frac{t - \varepsilon}{t} \right) = \tilde{\pi}_{\text{ISP}} \iff 0 > (2t - v)\varepsilon - \varepsilon^2. \quad (13)$$

Because $0 > 2t - v$ by assumption, the inequality on the right always holds, so that it is unprofitable for the ISP to set its hookup fee above $v - t$.

⁵We note that starting from a full coverage equilibrium with $x \geq 1/2$, when CP_A increases its price above $\bar{p}_A(x)$, x does not shift left toward zero because at $\bar{p}_B(x)$, the indifferent consumer receives zero utility from consuming content from CP_B and instead becomes indifferent between consuming from CP_B and exiting the market.

Step 3. By setting $\varepsilon = 0$ in Step 2 above, we can see that when $H = v - t$ in equilibrium, the equilibrium content subscription fees are given by $\hat{p}_A(x) = t(1 - x)$ and $\hat{p}_B(x) = tx$ under full coverage.⁶ At these prices, CP_A earns profit $\hat{\pi}_A(x) = t(1 - x)x$, which is maximized at $x = 1/2$ and leads to $\hat{\pi}_A(1/2) = t/4$.

However, in this case, for any $x > 1/2$, CP_A wishes to deviate to a higher price (and a partial coverage equilibrium). Under partial coverage, from Expression (6), we know that CP_A can earn $\tilde{\pi}_A = t/4$ by deviating to $\tilde{p}_A = t/2$, which is greater than $\hat{\pi}_A(x) = t(1 - x)x$ when $x > 1/2$. Similarly, we can show that CP_B would wish to deviate to a higher price for any $x < 1/2$. Thus, the equilibrium set of prices given in Proposition 1 is the unique full coverage subgame perfect Nash outcome. \square

Proof of Proposition 2.

Proof. The proof follows a similar approach to that of Proposition 1, but in two parts, one for $v \in [5t/3, 2t]$ and the other for $v > 2t$. For $v \in [5t/3, 2t]$, we proceed in two steps. In Step 1, we show that the highest hookup fee that maintains full coverage is an optimal response to CPs' equilibrium prices and that given the ISPs' optimal hookup fee, neither CP has an incentive to deviate to a price that leads to partial coverage. In Step 2, we show that there is a profitable deviation to partial coverage by either the ISP or one of the CPs from any other symmetric set of prices. For $v > 2t$, it suffices to show Step 1.

$v \in [5t/3, 2t]$, **Step 1.** Suppose that $v \in [5t/3, 2t]$. From the computation of partial equilibrium prices (Expressions (9) and (10)), we know that when $v = 5t/3$, $p_A^* = p_B^* = 2v/5 = v - t$. As the CP prices that prevail in the limit of the equilibrium under partial coverage as it approaches full coverage, $p_A^* = p_B^* = v - t$ represent a candidate pair of full coverage equilibrium prices. Under full coverage, given CP prices, the ISP maximizes H subject to $U_A(\cdot) = U_B(\cdot) = 0$, which in the case that $p_A^* = p_B^* = v - t$ leads to $H^* = v - (t + p_A + p_B/2) = t/2$. At $H^* = t/2$, the ISP has no desire to lower its price and brings about partial coverage by raising it. From Expression (5) it follows that the ISP's partial coverage demand when $p_A^* = p_B^* = v - t$ equals

⁶Suppose that $x > 1/2$. Substituting $\hat{p}_B(x) = tx$ into the objective function in Expression (8) and solving CP_A 's maximization problem for p_A , it becomes evident that the constraint binds—the unconstrained optimum price of $p_A(x) = t(x + 1)/2$ is strictly higher than $\hat{p}_A(x) = t(1 - x)$ for $x \geq 1/2$ and would lead the indifferent consumer with negative utility from consuming content from CP_A —and because the objective function is concave in p_A , $\hat{p}_A(x)$ is the best that CP_A can do under full coverage.

$2(t - H)/t$. Thus, the ISP maximizes its partial coverage profit by choosing $H = t/2$, which brings it back to the full coverage outcome.

We next want to know whether either CP has an incentive to deviate given its rival's price and the optimal ISP hookup fee. Suppose that $p_A = v - t + \varepsilon$ for $\varepsilon \neq 0$ given $p_B = v - t$. Note that CP_A raises its price if $\varepsilon > 0$ and lowers it if $\varepsilon < 0$. Suppose first that $p_A = v - t + \varepsilon$ and $p_B = v - t$ leads to full coverage, so that the ISP maximizes H subject to $U_A(\cdot) = U_B(\cdot) = 0$. Then, $\bar{\pi}_{ISP} = \bar{H} = (t - \varepsilon)/2$. By raising its price above $(t - \varepsilon)/2$, the ISP brings about partial coverage. Under partial coverage, the ISP solves:

$$\max_H H \left(\frac{2t - 2H - \varepsilon}{t} \right) \quad \text{s.t.} \quad x_A < x_B \quad (14)$$

which leads to a hookup fee of $\tilde{H} = t/2 - \varepsilon/4$. Observe that $\tilde{H} > \bar{H}$ only when $\varepsilon > 0$, implying that partial coverage does not emerge when $\varepsilon < 0$. We next consider the cases $\varepsilon > 0$ and $\varepsilon < 0$ in turn.

Suppose that $\varepsilon > 0$. The ISP's profit from deviating to $\tilde{H} = t/2 - \varepsilon/4$ equals $\tilde{\pi}_{ISP} = (2t - \varepsilon)^2/(8t)$. Thus, $\tilde{\pi}_{ISP} > \bar{\pi}_{ISP}$ holds if and only if $\varepsilon^2/(8t) > 0$ which is always the case. Thus, the ISP optimizes by deviating to partial coverage. Substituting $p_A = v - t + \varepsilon$ and $p_B = v - t$ into partial coverage profit Expression (6) yields $\tilde{\pi}_A = (2t - 3\varepsilon)(v - t + \varepsilon)/(4t)$. CP_A deviates from $p_A^* = v - t$ to $p_A = v - t + \varepsilon$ only if $\tilde{\pi}_A > \pi_A^* = (v - t)/2$. However, because $\pi_A^* > \tilde{\pi}_A$ if and only if $(3v - 5t)\varepsilon + 3\varepsilon^2 > 0$ and by assumption, $3v - 5t > 0$, CP_A does not wish to price higher than $p_A^* = v - t$. By the same logic, CP_B does not have a profitable deviation to a price higher than $v - t$.

Next, suppose that $\varepsilon < 0$. The lower CP_A price maintains full coverage and from $\tilde{H} < \bar{H}$ (for $\varepsilon < 0$) above, we know that the ISP would not wish to deviate to a higher price. Thus, when $\varepsilon < 0$, the ISP optimally sets its price to $\bar{H} = (t - \varepsilon)/2$ and achieves full coverage. To see whether CP_A has an incentive to deviate to begin with, we compare $\pi_A^* = (v - t)/2$ to $\bar{\pi}_A$, CP_A 's full coverage deviation profit. It can be verified that $\pi_A^* \geq \bar{\pi}_A$ holds if and only if $\varepsilon \leq 2t - v$. Because $v \in [5t/3, 2t]$ and $\varepsilon < 0$, π_A^* is always greater than or equal to $\bar{\pi}_A$, so that neither CP has a profitable deviation. Therefore, we have shown that $p_A^* = p_B^* = v - t$ and $H^* = t/2$ represent a full coverage equilibrium outcome for $v \in [5t/3, 2t]$. However, when $v \geq 2t$, there is always an ε close enough to zero such that each CP wishes to deviate unless

$v = 2t$. Because $v - t = t$ when $v = 2t$, $p_A^* = p_B^* = t$ is our symmetric equilibrium candidate for $v > 2t$.

$v \in [5t/3, 2t]$, **Step 2.** Suppose instead that $p_A^* = p_B^*$ at some price other than $v - t$. Suppose first that $p_A^* = p_B^* = v - t + \varepsilon/2$ for some $\varepsilon > 0$. Then, the best that the ISP can do under full coverage is $H = (t - \varepsilon)/2$. By deviating to a higher price, the ISP causes partial coverage. In this case, the ISP solves Expression (14), which leads to a hookup fee of $H = t/2 - \varepsilon/4$. From Step 1, we know that this will be more profitable for the ISP, so that $p_A^* = p_B^* = v - t + \varepsilon/2$ cannot be a full coverage outcome when $\varepsilon > 0$.

Next, suppose that $p_A^* = p_B^* = v - t + \varepsilon$ for some $\varepsilon < 0$ leads to full coverage. Suppose that CP_B deviates to $\bar{p}_B = v - t$. From Step 1, we know that in this case, under full coverage, the ISP sets $\bar{H} = (t - \varepsilon)/2$ and does not wish to deviate to partial coverage. Comparing CP profit in the proposed equilibrium, $\pi_B^* = (v - t + \varepsilon)/2$, to its deviation profit, $\bar{\pi}_B = (v - t)(\varepsilon + t)/2t$, we have $\bar{\pi}_B \geq \pi_B^*$ if and only if $2t \geq v$, which holds by assumption. Thus $p_A^* = p_B^* = v - t$ is the only symmetric full coverage outcome for $v \in [5t/3, 2t]$.

$v > 2t$. It remains to show that $p_A^* = p_B^* = t$ and $H^* = v - 3t/2$ represent a full coverage equilibrium outcome for $v > 2t$. We first consider the ISP's deviation incentive. A lower H is clearly suboptimal whereas a higher H leads to partial coverage. In the latter case, when $p_A^* = p_B^* = t$, the ISP maximizes $H(2v - 2t - 2H)/t$ with respect to H , which yields $\tilde{H} = (v - t)/2$. However, $\tilde{H} < H^*$ whenever $2t < v$, meaning that the ISP does not have a profitable deviation to partial coverage. Moreover, from Equations (2) and (3) in the game where the ISP moves first, we know that $p_A = p_B = t$ represents a set of mutual CP best responses, so that $p_A^* = p_B^* = t$ and $H^* = v - 3t/2$ indeed represent a full coverage equilibrium in the game where the CPs move first whenever $v \geq 2t$. \square