# Quality Differentiation and Optimal Pricing Strategy in Multi-Sided Markets*

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**Abstract**

This paper analyzes the generalized quality differentiation model in multi-sided markets with positive externalities, which leads to new insights into the optimal pricing structure of the firm. We find that quality differentiation for users on one side leads to a decrease in the price charged to users on the other side, thereby affecting the pricing structure of multi-sided firms. Also, quality differentiation affects the strategic relationships among the choice variables for the platform, so that the platform strategically uses it to raise its profits.

**Keywords**: Multi-Sided Market; Quality Differentiation; Platform Business Strategies

**JEL Classification Numbers:** D43; L11; L42

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# 1 Introduction

There has been an increasing shift toward multi-sided firms in many industries. The early, pioneering models of multi-sided platforms were introduced by Armstrong (2006), Caillaud and Jullien (2003), Parker and Van Alstyne (2005), and Rochet and Tirole (2003). It is important to recognize that many real-world organizations determine how close or how far they are from a multi-sided economic model, based on changing industrial parameters. In this paper, we examine how platforms in multi-sided markets optimally use quality differentiation as a business strategy.

As many previous studies emphasize, it is important to see how multi-sided platforms differ from typical one-sided firms and how such differences lead to new business implications. Many markets that have traditionally been one-sided now feature more two-sided firms due to advanced technology; for example, the taxi industry only had one-sided firms before Uber appeared. It is usually observed that firms begin with an one-sided model and switch to a multi-sided model as they become more established. Doing so allows potential platforms to overcome the "chicken-and-egg" problem by first providing complementary goods themselves. For instance, Amazon started off as a pure retailer but has moved closer to a two-sided model over time by enabling third-party sellers to trade directly with consumers on its website. We can also find many other examples in which each firm faces a strategic choice of how many sides to pursue. For instance, in the personal computer market, Apple produces its own hardware, whereas Microsoft leaves this to independent manufacturers. As a result, Apple manages only a two-sided platform with consumers and software providers, while Microsoft manages a three-sided platform with consumers, software providers, and hardware providers. Indeed, there is an increasing shift from traditional business structures, with the firm focusing only on one side of users, to platforms where the firm serves more than one side of users.

Given this shift toward multi-sided business, the main strategic choice, which we analyze in this paper, is the choice of the quality of the interaction between users on two different sides. We focus on platforms that enable interaction between two sides, for example, buyers and sellers. The term "interaction" covers various traditional interactions, such as those observed in auction houses, and those on internet sites in person-to-business transactions, for example, Amazon. This also includes interactions or exchanges between application developers and application users on software platforms such as computers (e.g., Apple, Microsoft); mobile devices (e.g., iPhone, Samsung); and video games (e.g., Sony PlayStation, Xbox). These markets usually have more than one type of quality access on the buyers' side. For example, Amazon offers buyers two types of quality access; the basic (low-quality) access is free, while premium access (*Prime membership*, which is high quality) is the paid service. It is easy to see that the interactions or exchanges through the high-quality access have better quality—for example, *Prime* gives you two-day shipping. Another example is Apple App Store on iPhone because it acts as a medium between app users and app developers. Apple quality differentiates on the buyers' side by offering two qualities, such as the iPhone 6 and iPhone 6 plus. These two phones differ in size and camera resolution. This affects the utility of buyers from apps on the phone: iPhone users gain higher utility from apps such as Netflix and Snapchat on the iPhone 6 plus than on the iPhone 6. These are examples of quality differentiation by firms serving two sides. Ultimately, this paper provides business implications of quality differentiation in the multi-sided platform market by building a simple model of two-sided monopolists in a market with interactions between buyers and sellers. The firm chooses the price and the quality of interaction for the buyers' side.

The first main finding of the paper is that a firm provides higher quality per dollar to buyers as it serves more sides. That is, when a firm expands its business from one-sided service (serving buyers/app users only) to multi-sided service as a platform (serving both buyers/app users and sellers/app developers), it provides better quality to buyers. Intuitively, if a firm offers a better quality-price menu to buyers, it attracts more buyers. When it serves seller's side at the same time, more demand from buyer's side makes sellers on the othe side earn more revenue. The platform can extract those additional seller's side revenues, which incentivizes it to offer a better quality-price ratio to buyers.

We also find that the quality differentiation on the buyers' side decreases the price charged on the sellers' side compared to the price charged by a single-quality, two-sided firm. The intuition behind this finding is related to product differentiation and demand elasticity. If the platform provides multiple quality options for buyers, it means that it provides more differentiated products, which makes buyers more inelastic to price changes because the platform can extract extra surplus from high-quality buyers without deterring buyer participation. As the buyers' side becomes more inelastic, it creates an incentive for the platform to extract a higher rent from the buyer side and subsidize the seller side by lowering the price faced by sellers.

Such quality differentiation also leads to a much greater profit for the platform. By providing more quality choices, even if the quality differentiation is small, the platform is able to obtain more profits. Given that each buyer is heterogeneous with respect to his valuation of product quality, there is an incentive for the platform to offer different levels of product quality at different prices to extract more rents from buyers. Quality differentiation not only increases the buyers' market size, which increases the extensive margin, but also affects the platform's intensive margin. One of the model predictions suggests that the platform is able to earn higher extra markup from the high-valuation group of buyers (who will buy a high-quality product) if the quality gap is widened. Based on the model predictions, we discuss some business implications.

**Related literature** There is a broad literature on corresponding problem of a monopolistic firm seeking to maximize profits by offering quality-differentiated products in one-sided standard markets. The seminal papers are Spence (1977), Mussa and Rosen (1978), Maskin and Riley (1984), and M. Itoh (1983). We generalize this problem by varying the number of sides served by the firm.

This paper is also related to the literature on pricing structure in markets with multi-sided firms (e.g., Rochet and Tirole (2003), Armstrong (2006b), and Reisinger (2010)). Aside from being broadly related to the literature on pricing in multi-sided markets, papers on skewed pricing in multi-sided markets are closely related to our paper in terms of our theoretical implications. Suarez and Cusumano (2008) discuss the platform's subsidy pricing strategy to attract greater user adoption, although they do not set up an economic model to confirm this strategy. Bolt and Tieman (2008), Schmalensee (2011), Dou and Wu (2018) study skewed pricing strategies in two-sided markets, i.e., the subsidy and money sides. However, those papers do not consider forms of product differentiation, such as the quality differentiation examined in our paper, as a means of skewing prices.

Regarding markets with multi-sided firms, few papers have focused on firms choosing the quality of interaction on their platform. These papers study markets with negative network externalities and do not endogenize the quality choice. Crampes and Haritchabalet (2009) examine the choice of offering pay ads regime and no pay ads packages. Peitz and Valletti

(2004) compare the advertising intensity when media operators offer free services and when the subscription price is positive. Viecens (2006) is an exception because she studies a setup with endogenous quality differentiation on two-sided platforms. However, the quality differentiation in her model takes a different form from ours in that hers focuses on the quality provided by users on one side and not on the quality provided by the platform itself. Therefore, her results do not provide any implications for the platform's dynamic pricing structure, such as subsidizing one side at the expense of the other. Another quality-related aspect explored in the context of two-sided markets is the case in which users care about the quality of the other users with whom they interact. These papers are relevant for matching markets such as dating sites. Jeon et al. (2016) examine this problem in a platform setting. Renato and Pavan (2016) consider this problem in a matching setup. Hagiu (2012) studies a model in which users value the average quality of other users. The setup in these papers, however, is different from ours in that we focus on the quality of interaction and not on the quality of users.

Our analyses are also related to the literature on product differentiation in multi-sided markets, in that quality differentiation is one form of product differentiation. Smet and Cayseele (2010) focus on product differentiation in platform markets, which still differs from our paper in that they do not account for its consequences for optimal pricing strategies.

To the best of our knowledge, the literature has not focused on endogenizing the choice of network quality in multi-sided markets with positive externalities. Therefore, our results on how multi-sided markets combined with quality differentiation on a platform provide new insights into the related business.

## 2 Model Setup

We model the interaction between buyers and sellers. Economic value is created through the interaction between these two sides. We consider the case of a single firm providing a platform for interactions between buyers and sellers. For instance, the Apple App Store on iPhone devices acts as a platform for interactions between app developers (sellers) and app users (buyers). In addition to charging an access fee to use the platform, the firm can also control the quality of interaction. We begin considering a model that omits the practice of quality differentiation as a benchmark and then modify the model to allow for its practice.

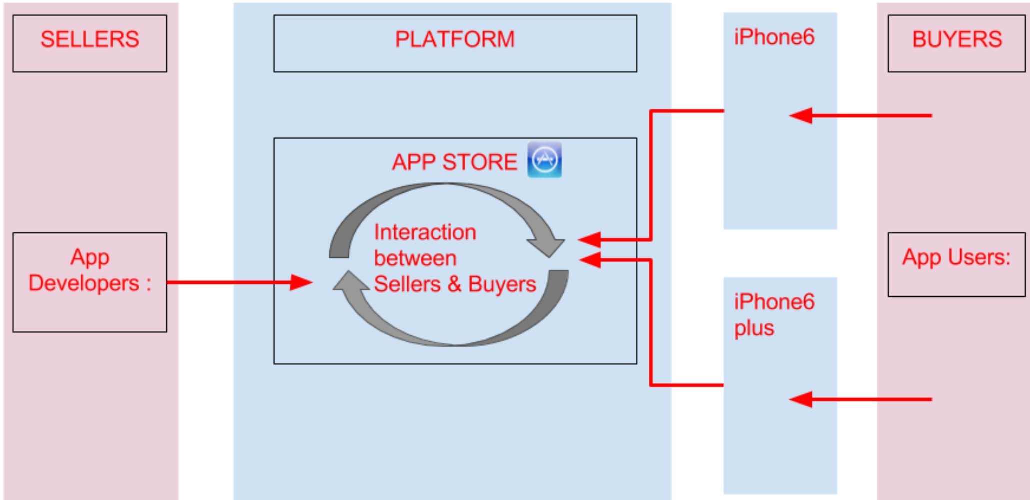


Figure 1: Two-sided firm with quality differentiation

Figure 1 displays the structure of the two-sided firm with quality differentiation on the buyers' side. It displays the Apple App Store as a case of a two-sided firm with quality differentiation on the buyers' side. The App Store acts as a platform for interactions between app developers (sellers) and app users (buyers). The two types of access quality offered on the buyers' side are the iPhone 6 and iPhone 6 plus—the iPhone 6 plus is the higher quality access, as it has better resolution and a larger display.

Specifically, we model a two-sided monopolist firm that price discriminates on the buyers' side by offering two different types of access quality: $q_k \in \{q_l, q_h\}$ where $k$ denotes the quality provision, either $l$ow or $h$igh. In this model, the users' gain from the platform comes through interaction between end users. The quality variable controls the gain from each such interaction.

The two sides are buyers and sellers. Both buyers and sellers obtain utility from interacting with each other. We assume that both buyers and sellers are heterogeneous with respect to the per interaction or usage benefit. The usage or per interaction benefits are $b_i^b(q_k)$ for the buyer side (for individual buyer $i$) and $b_j^s$ for the seller side (for individual seller $j$) where the superscript $b$ ($s$) denotes buyers (sellers). For simplicity, we assume that $b_i^b(q_k) = B\alpha_i^b q_k$, where $B$ represents the basic benefit for every buyer. This benefit is dependent on the quality of interaction; thus, the monopolist can control the benefit by choosing the quality of platform access, which is denoted $q_k \in [q_h, q_l]$ on the buyers' side. The monopolist uses this quality instrument, $q_k$, to influence the benefit derived from different qualities of platform access.

Additionally, each buyer pays a price for using the platform, which is denoted $p_k^b$ as a usage or per transaction fee. The term $\alpha_i^b$ denotes the heterogeneity among buyers; it follows a distribution function $F^b(.)$ with support on $[0, 1]$ and density is $f^b(.)$. For a buyer $i$, the utility function is given by:

$$\begin{aligned} U_{ik}^b &= [b_i^b(q_k) - p_k^b]N^s, \quad \text{where } k \in \{l, h\} \\ \Leftrightarrow U_{ik}^b &= (B\alpha_i^b q_k - p_k^b)N^s, \end{aligned} \tag{1}$$

where $p_k^b$ is the usage or per transaction price. The buyers' utility is the net benefit from each interaction with the other side, i.e., $(B\alpha_i^b q_k - p_k^b)$ multiplied by the number of interactions, which is denoted $N^s$.[1] From the utility specification, it can be shown that the buyer with the highest benefit is that with $\alpha_i^b = 1$, $B\alpha_i^b = B$.

The sellers' side is not affected by quality; therefore, there is no quality component. For simplicity, we assume that $b_j^s = S\alpha_j^s$, where $\alpha_j^s$ represents seller $j$'s heterogeneity with respect to the per usage benefit, which is also distributed by a distribution function $F^s(.)$ with support on [0,1] and density is $f^s(.)$. For seller $j$, the utility function is given by:

$$\begin{aligned} U_j^s &= (b_j^s - p^s)N^b \\ \Leftrightarrow U_j^s &= (S\alpha_j^s - p^s)N^b. \end{aligned} \tag{2}$$

The sellers' utility is the net benefit from each interaction with the other side, i.e., $S\alpha_j^s - p^s$ multiplied by the number of interactions, which is denoted $N^b$. All sellers are charged $p^s$ per interaction, so the price discrimination is only on the buyer side.[2] Additionally, we assume that the total number of interactions is $N^bN^s$.[3]

Next, the cost of the two-sided monopoly firm depends on the quality provided. The total cost of a transaction is given by $c(q_l) \geq 0$ for a transaction between a low-quality buyer and a seller and $c(q_h) \geq 0$ for a transaction between a high-quality buyer and a seller.[4] The cost

[1] We assume here that every buyer interacts with every seller and that every seller interacts with every buyer. We can easily extend this to a model where the number of interactions is a function of the total number of sellers, such as $g(N^s)$. The results are robust to such extensions.

[2] In this model, we assume that the seller side is affected only by the total number of buyers, not the type of buyer with which a seller interacts. Thus, the total number of interactions for each seller is $N^b$, and the total number of interactions by sellers is not affected by the type of buyer with which they interact.

[3] Note that the qualitative results still hold under a generalized setup with any function of $N^bN^s$.

[4] For simplicity, we normalize the cost for sellers $c^s = 0$.

function is assumed to be increasing and convex in quality ($c'(q) > 0$ , $c''(q) > 0$). We analyze the non-trivial case in which $q_h > q_l$.

The demand on buyers and sellers' sides is represented by $D^b$ and $D^s$, respectively. In equilibrium, demand will be equal to the number of participants on each side, which means $D^b_k = N^b_k$ and $N^s_k = D^s_k$ where $k \in \{l, h\}$. Given the equilibrium demands, we turn to the monopolistic platform's problem. We restrict our attention to the case in which the platform charges a per interaction price.[5] The monopoly platform's problem can be written as follows:

- If one type of quality is offered:

$$\max_{(p^s, p^b, q)} \Pi = [p^b + p^s - c(q)] D^b D^s. \tag{3}$$

- If two types of quality are offered:

$$\max_{(p^s, p^b_l, p^b_h, q_l, q_h)} \Pi = [p^b_l + p^s - c(q_l)] D^b_l D^s + [p^b_h + p^s - c(q_h)] D^b_h D^s. \tag{4}$$

Note that if the monopolist serves only one side of the market, say the buyer's side, its profit function is given as follows:

$$\max_{(p^b, q)} \quad \Pi_{\text{one-sided monopolist}} = [p^b - c(q)] D^b, \tag{5}$$

in the case of one quality. Throughout the paper, we make the following assumptions.

**Assumption 1.** The cost is increasing and convex in quality : $c'(q) > 0$ , $c''(q) > 0$.

This suggests that it becomes increasingly costly to provide higher quality service, which is a standard assumption.

**Assumption 2.** $f^b(1) > 0$.

This implies that we consider the case with a positive mass of consumers.

**Assumption 3.** $\frac{\partial U^b(\alpha^b_i, p^b, q)}{\partial p^b} < 0$, $\frac{\partial U^b(\alpha^b_i, p^b, q)}{\partial q} > 0$, $\frac{\partial U^s(\alpha^s_i, p^s)}{\partial p^s} < 0$.

**Assumption 4.** The distribution functions for buyers and sellers are increasing in type:

[5] However, this can be generalized to the case in which the platform charges per interaction and a membership fee as in $p = P_{\text{per transaction}} + \frac{P_{\text{membership fee}}}{N}$.

$(f^b)' \geq 0$ and $(f^s)' \geq 0$.

The more benefit users obtain per usage, the more users use the platform.

**Assumption 5.** The inverse hazard rate is non-increasing in user type: $\frac{\partial \frac{1-F^k(\theta)}{f^k(\theta)}}{\partial \theta} \leq 0, k = \{b, s\}$.

As users value the per usage benefit more, there are more platform users than non-users.

**Assumption 6.** The single crossing property holds: $\frac{\partial^2 U^b(\alpha_i, q(\alpha_i))}{\partial \alpha_i \partial q} > 0$.

We can always distinguish high-type buyers from low-type buyers based on their $\alpha_i$.

# 3 Model

## 3.1 Model with one quality case

As a benchmark, we first analyze the case in which only one quality is offered to buyers. The two-sided monopolist offers a single quality of access in this case. Buyers have the following preferences:

$$U_i^b = B\alpha_i^b q - p^b, \tag{6}$$

where $q \in [0, 1]$ denotes the quality index. The seller has the following utility:

$$U_i^s = S\alpha_i^s - p^s. \tag{7}$$

We start by analyzing the user side (buyers and sellers) to identify the equilibrium demand. The equilibrium demand functions are derived from the participation constraint:

$$\begin{aligned} D^b &= Prob\left(U_i^b \geq 0\right) \quad \Rightarrow D^b = 1 - F^b\left(\frac{p^b}{Bq}\right) \\ D^s &= Prob\left(U_j^s \geq 0\right) \quad \Rightarrow D^s = 1 - F^s\left(\frac{p^s}{S}\right) \end{aligned} \tag{8}$$

The monopoly problem can be written as follows:

$$\max_{(p^s, p^b, q)} \Pi = [p^b + p^s - c(q)]D^b D^s. \tag{9}$$

We can solve for the optimal solutions for the buyer's side and seller's side as follows:

$$\frac{p^b - c(q) + p^s}{p^b} = \frac{1}{\epsilon^b}; \quad \frac{p^s - c(q) + p^b}{p^s} = \frac{1}{\epsilon^s}. \tag{10}$$

Equation (10) is similar to the standard Lerner pricing formula, which states that the markup on price is equal to the inverse of the price elasticity of demand. The price elasticity of demand for buyers (sellers) is represented by $\epsilon^b$ ($\epsilon^s$). The added term in the case of a two-sided firm is the extra opportunity cost of increasing the price on the buyers' (sellers') side, which is the marginal loss on the sellers' (buyers') side, equal to $p^s$ ($p^b - c(q)$).

Finally, the optimal condition for quality is given as follows:

$$c'(q) = \frac{[p^b + p^s - c(q)]}{D^b}\frac{\partial D^b}{\partial q}. \tag{11}$$

The optimal quality equates the marginal cost to the marginal revenue of increasing quality. The marginal revenue is the change in buyers' demand,[6] which is represented by $(D^b)'_q$, times the per transaction profit. From the conditions given above, the equilibrium price-quality structure is given by the following equation:

$$\frac{qc'(q)}{\nu^b} = \frac{p^b}{\epsilon^b} = \frac{p^s}{\epsilon^s}, \tag{12}$$

where $\nu^b$ is the quality elasticity of demand on the buyers' side.

## 3.2 Model with two qualities case

We start by analyzing the users' side (buyers and sellers) to identify the equilibrium demand. First, the buyers have two choices for accessing the platform. They can join the platform through either low-quality access or high-quality access. Given the two types of quality, high and low, the number of participants joining with low-quality access is determined by the number of buyers who satisfy the following two conditions:

1. (IR constraint) The buyers' utility from low-quality access is greater than zero: $Pr(U_l^b \geq 0)$.

[6] In the optimal quality equation, the term for the number of sellers ($D^s$) is canceled out, as it is present on both sides of the equation.

2. (IC constraint) Buyers for whom the utility derived from low-quality access exceeds that from high-quality access: $Pr(U_l^b \geq U_h^b)$.

The two conditions jointly determine the proportion of low-type buyers.

$$D_l^b = \Pr\left(\frac{p_h^b - p_l^b}{B(q_h - q_l)} \geq \alpha_i^b \geq \frac{p_l^b}{Bq_l}\right) = F^b\left(\frac{p_h^b - p_l^b}{B(q_h - q_l)}\right) - F^b\left(\frac{p_l^b}{Bq_l}\right), \tag{13}$$

where $D_l^b \equiv D^b(p_h^b, p_l^b, q_h, q_l)$. The number of participants joining the high-quality service is given by the number of buyers who satisfy the following two conditions:

1. (IR constraint) The buyers' utility from the high-quality good is greater than zero: $Pr(U_h^b \geq 0)$.
2. (IC constraint) Buyers for whom the utility derived from the high-quality good exceeds that from the low-quality good: $Pr(U_h^b \geq U_l^b)$.

The IR condition is satisfied when the IC constraint of the high type and IR constraint of the low type holds.[7] Thus, the proportion of high-type buyers is given by:

$$\begin{aligned} D_h^b &= \Pr(U_h^b \geq U_l^b) \\ \Leftrightarrow D_h^b &= \Pr\left(\alpha_i^b \geq \frac{p_h^b - p_l^b}{B(q_h - q_l)}\right) = 1 - F^b\left(\frac{p_h^b - p_l^b}{B(q_h - q_l)}\right), \end{aligned} \tag{14}$$

where $D_h^b \equiv D^b(p_h^b, p_l^b, q_h, q_l)$. Given the utility function for buyers, the total number of buyers joining the platform is given by:

$$\begin{aligned} D^b &= \Pr(U_l^b \geq 0) = \Pr(b_l^b \geq p_l^b) \\ \Leftrightarrow D^b &= \Pr(B\alpha_i^b q_l \geq p_l^b) = \Pr(\alpha_i^b \geq \frac{p_l^b}{Bq_l}) = 1 - F^b\left(\frac{p_l^b}{Bq_l}\right), \end{aligned} \tag{15}$$

where $D^b \equiv D(p_l^b, q_l)$. Equation (15) shows us how the number of participants on the buyer side depends only on the price and quality of the low-quality good. Although there are network externalities in the total utility derived from the platform or the gross transaction utility $[b^b(q_k) - a^b(q_k)]N^s$, the per unit transaction demand is not dependent on the participation rate

[7] We maintain the standard single crossing condition, which implies that higher types have greater willingness to pay (WTP) for quality at any price or that consumers may be ordered by their type.

on the other side.[8] This is because the participation constraint (IR constraint) for the high-quality buyers is slack. This means that the participation of low-quality buyers guarantees the participation of high-type buyers. In other words, the buyers on the margin of joining the platform are low-quality buyers.

Next, the total number of sellers who join the platform is given by:

$$D^s = \Pr(U^s \geq 0) = 1 - F^S\left(\frac{p^s}{S}\right), \tag{16}$$

where $D^s \equiv D^s(p^s)$. Given the total number of buyers and sellers, the equilibrium level of participation is:

$$\begin{aligned} D^b &= D(p_l^b, q_l) = 1 - F^b\left(\frac{p_l^b}{Bq_l}\right) \\ D_h^b &= D^b(p_h^b, p_l^b, q_h, q_l) = 1 - F^b\left(\frac{p_h^b - p_l^b}{B(q_h - q_l)}\right) \\ D^s &= D^s(p^s) = 1 - F^s\left(\frac{p^s}{S}\right). \end{aligned} \tag{17}$$

Given quality differentiation, the monopoly problem can be written as follows:

$$\max_{(p^s, p_l^b, p_h^b, q_l, q_h)} \Pi = [p_l^b + p^s - c(q_l)]D^b D^s + [p_h^b + p^s - c(q_h)]D_h^b D^s. \tag{18}$$

The following is the breakdown of the equilibrium prices and quality:

### 3.2.1 Price of low-quality access on the buyer side

We first examine the price of low-quality access on the buyer side.

$$p_l^b = \underbrace{c(q_l) + \frac{D_l^b}{(-D_l^b)'_{p_l^b}}}_{\text{cost plus classic market power}} - \underbrace{\left[1 + \frac{(D_h^b)'_{p_l^b}}{(D_l^b)'_{p_l^b}}\right] p^s}_{\text{effect on the sellers side}} - \underbrace{\frac{(D_h^b)'_{p_l^b}}{(D_l^b)'_{p_l^b}}[p_h^b - c(q_h)]}_{\text{switching effect}} \tag{19}$$

$$\Leftrightarrow \quad \frac{p_l^b - c(q_l) + p^s}{p_l^b} = \frac{1}{\epsilon_l^b}, \text{ after using equilibrium value of } p_h^b.$$

[8] This setup has one restriction that we need to impose, which is that the proportion of low-type buyers has to be non-negative: $D_l^b \geq 0$.

Equation (19) is similar to the standard Lerner pricing formula, which states that the markup on price is equal to the inverse of the price elasticity of demand. The price elasticity of demand for low-quality access is represented by $\epsilon_l^b$. Please note that the price elasticity of demand is with respect to the total demand ($D^b$) on the buyers' side.

### 3.2.2 Price of high-quality access on the buyer side

The price of high-quality access on the buyer side can be obtained as follows:

$$p_h^b = p_l^b + \underbrace{[c(q_h) - c(q_l)] + \frac{p_h^b}{\epsilon_h^b}}_{\text{additional cost plus extra market power}} . \tag{20}$$

The optimal price for high quality is equal to the price for low-quality access and the additional cost plus an additional markup $\frac{p_h^b}{\epsilon_h^b}$, where $\epsilon_h^b$ is the price elasticity with respect to demand for high-quality access.

### 3.2.3 Price for sellers

We now turn our attention to the price for sellers.

$$\frac{p^s - c(q_l) + \left[\frac{D_h^b}{D^b}\frac{D_h^b}{(-D_h^b)'_{p_h^b}} + p_l^b\right]}{p^s} = \underbrace{\frac{1}{\epsilon^s}}_{\text{Classic mark-up}} , \tag{21}$$

where $(D_h^b)'_{p_h^b}$ means that $\frac{\partial D_h^b}{\partial p_h^b}$. Again, Equation (21) is similar to the standard Lerner pricing formula, which states that the markup on price is equal to the inverse of the price elasticity of demand. The added term in the case of a two-sided firm is the extra opportunity cost of increasing the price on the sellers' side, which is the marginal loss on the buyers' side and equal to the average per interaction profit on the buyers' side.[9]

### 3.2.4 Low-quality service for buyers

Given prices, the monopolistic platform solves the profit maximization problem separately for the low- and high-quality service for buyers. First, for low-quality service, the first order

[9]Note that we have normalized the cost on the seller's side to zero, so $c^s = 0$.

condition can be derived as follows:

$$\underbrace{c'(q_l)}_{\text{MC}} + \underbrace{\frac{(-D_h^b)'_{q_l}}{D_l^b}[p_h^b - c(q_h)]}_{\text{MC from switching effect}} = \underbrace{\frac{(D_l^b)'_{q_l}}{D_l^b}[p_l^b - c(q_l)]}_{\text{MR from low quality service buyers}} + \underbrace{\left[\frac{(D^b)'_{q_l}}{D_l^b}\right]p^s}_{\text{MR from the seller's side}}, \tag{22}$$

where $(D_k^b)'_{q_k}$ means that $\frac{\partial D_k^b}{\partial q_k}$ and $(D_k^b)'_{q_j}$ means that $\frac{\partial D_k^b}{\partial q_j}$ where $j \neq k$. The marginal cost is the marginal increase in cost, $c'(q_l)$, and the loss of high-type buyers as buyers switch to low quality, $\frac{(-D_h^b)'_{q_l}}{D_l^b}[p_h^b - c(q_h)]$. The marginal revenue is the gain in low-quality buyers, $\frac{(D_l^b)'_{q_l}}{D_l^b}[p_l^b - c(q_l)]$, and the gain from the network effect on the sellers' side, $\frac{(D^b)'_{q_l}}{D_l^b}p^s$.

### 3.2.5 High-quality service for buyers

Here, the first order condition for the high-quality service for buyers is given by:

$$\underbrace{c'(q_h)}_{\text{MC}} = \underbrace{\frac{(D_h^b)'_{q_h}}{D_h^b}\left\{[p_h^b - c(q_h)] - [p_l^b - c(q_l)]\right\}}_{\text{MR from switching to high quality service}} \tag{23}$$

The marginal cost $c'(q_h)$ should be equal to the marginal revenue, which is the product of increased high-quality buyers $\frac{(D_h^b)'_{q_h}}{D_h^b}$ and the extra markup generated from the increase in buyers $[p_h^b - c(q_h)] - [p_l^b - c(q_l)]$.

## 4 Equilibrium Results

We derive several important implications from the model. Before we look into the diverse effects of quality differentiation on the platform's strategies, we first address that the monopolistic firm provides higher quality to buyers for each dollar that they pay because it serves more sides of the market. As the monopolist opens more sides to serve, say from a one-sided firm serving buyers only to a two-sided firm as a platform serving both buyers and sellers, such openness increases the quality per dollar offered to buyers. Mathematically, the quality per dollar is denoted as $\frac{q}{p^b}$. This finding is summarized in Proposition 1.

**Proposition 1.** *As a firm serves more sides of the market, it provides higher quality per dollar offered to buyers:* $\frac{q}{p^b}^{\text{One-sided firm}} \leq \frac{q}{p^b}^{Two-sided firm}$.

Intuitively, the monopolist serving multiple sides has more incentive to offer a better quality-price ratio to buyers because it now obtains more profit from the seller's side. Two-sided firm as a platform offer buyers a better quality-price menu, which attracts more buyers. Ultimately, the monopolist reaps the benefit of more revenue from seller's side.

Returning to the multi-sided platform case, we first find that quality differentiation reduces the other side's price level. The Appendix contains a proof of this result.

**Proposition 2.** *Under stronger conditions for cost, quality differentiation on the buyers' side decreases the price charged on the sellers' side relative to the price charged by the platform that offers one quality.*

The above result holds under the following assumptions.

- A.1: Non-decreasing ratio of marginal cost to quality

$$\frac{\partial[\frac{c'(q)}{q}]}{\partial q} \geq 0. \tag{24}$$

- A.2: Condition on buyers' distribution, $\alpha_i^b \sim F^b(.)$

$$\frac{\partial[\frac{f^b(\theta)}{\theta}]}{\partial \theta} \geq 0. \tag{25}$$

The first condition implies that as quality increases, the increase in marginal cost is more than the increase in quality. This imposes a higher cost for increasing quality. Thus, under this condition, the difference between the two quality levels $(q_h - q_l)$ will be relatively small. The second condition implies that there are more buyers that enjoy a large benefit from joining the platform.

Quality differentiation on the buyers' side has two effects, the demand effect and the information rent effect. The demand effect of quality differentiation decreases the opportunity cost of increasing demand. This is because under quality differentiation, the firm only has to decrease the profit from the marginal buyers, namely the low-quality buyers, to increase demand. On the other hand, the information rent effect enables the firm to become more effective at extracting rent from the high-type buyers.

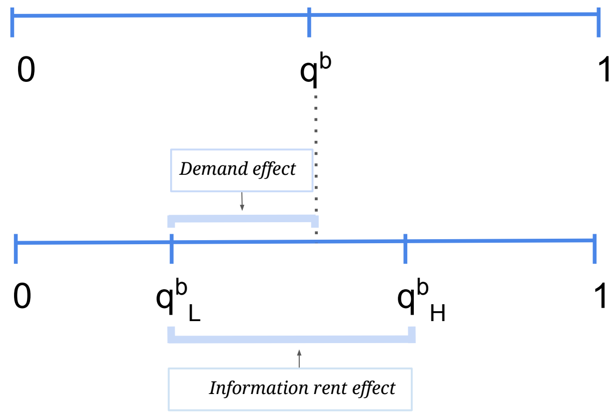


Figure 2: Effect of quality differentiation

Under $A.1$ and $A.2$, the demand effect dominates the information rent effect. Consequently, under the above assumptions, by quality differentiation, the participation on the buyers' side becomes less elastic to platform pricing since the platform can extract additional surplus from high-quality buyers without deterring buyers' participation.

Thus, the effect of offering low- and high-quality access to the buyers makes the buyers more inelastic. Now, recall that for multi-sided firms, the optimal pricing scheme is to subsidize the more elastic side of the market and extract rents from the other, more inelastic, side. As the buyers' side becomes more inelastic, it creates an incentive for the platform to extract higher rent from the buyers' side and subsidize the sellers' side by lowering the price charged to sellers.

This can be explained by comparing the equilibrium pricing structures of the two cases:

$$\begin{aligned} &\textit{one-quality case} \rightarrow \frac{p^s}{\epsilon^s} = \frac{p^b}{\epsilon^b} \\ &\textit{two-quality case} \rightarrow \frac{p^s}{\epsilon^s} = \frac{p_l^b}{\epsilon^b} + \underbrace{\frac{D_h^b}{D^b}\frac{p_h^b}{\epsilon_h^b}}_{\text{extra markup from quality differentiation}} \end{aligned} \tag{26}$$

Similar to the single-quality case, the platform faces a tradeoff of whether to charge a higher price on the buyer or seller side in cases with two quality types. After quality differentiation, the tradeoff features an additional markup benefit on the buyers' side, namely, the extra margin from high-quality buyers. Thus, the monopolist becomes more efficient in extracting rent from the buyers' side. Given this, the platform has a higher incentive to increase the demand for its platform by lowering the price on the sellers' side.

We also find that quality differentiation leads to greater profit for the platform, as in Proposition 3. The detailed proof is in the Appendix.

**Proposition 3.** *The platform strictly prefers to price discriminate by quality on the buyers' side*

Proposition 3 implies that a platform that provides only one type of quality is able to obtain more profit if it slightly differentiates product quality. Even a minor quality improvement with a small price increase can increase the platform's profit as long as it continues to provide differentiated products, such as low- and high-quality products.

Next, we analyze how the pricing strategy for the buyers' side under quality differentiation is affected by the sellers' side equilibrium. We show that the price difference between the low- and high-quality products depends on two factors: the cost difference and the extra markup from quality differentiation. However, how much more the platform can charge buyers for the high-quality product depends only on buyer side variables, not those on the sellers' side.

We also find that the extra markup from quality differentiation is increasing in the quality gap. Thus, if the platform sufficiently differentiates its product line with respect to quality, it can earn higher markup. Proposition 4 summarizes this finding.

**Proposition 4.** *The difference between the prices of low- and high-quality products does not depend on the sellers' side equilibrium, which is* $p_h^b - p_l^b = c(q_h) - c(q_l) + \underbrace{p_h^b/\epsilon_h^b}_{\text{extra mark up due to difference in quality}}$.

Proposition 4 is intuitive and fits with reality. The quality differentiation in our context is implemented by the platform itself; it has nothing to do with any product differentiation conducted by sellers on the platform. In other words, sellers provide the same lines of products to all buyers on the platform regardless of which quality of the platform's service the latter use. For instance, any seller's product on Amazon is accessible to consumers of both Amazon's basic service and Amazon Prime. When Amazon decides how much more to charge for Prime membership, it considers only (1) how much it costs more to provide two-day shipping for Prime members and (2) how much additional market share it earns from providing a different quality.

Next, we examine how the platform's optimal quality provision is related to the seller side equilibrium. By comparing Equations (22) and (23), we derive the following proposition.

**Proposition 5.** *The platform's optimal quality level for the high-quality service does not depend on the seller side equilibrium, whereas that for low-quality service increases in the marginal revenue from the seller side.*

Proposition 5 states that the platform has an incentive to increase the quality level of the basic service (low quality) because of the positive network effect coming from the seller side, whereas it does not consider the network effect when determining the optimal quality level of the premium (high-quality) service. The intuition behind this finding relates to total buyer demand. As in Equation (15), the participation constraint for the high-quality buyers is slack, which means that the effect of increasing the quality level of the premium service does not increase the total buyer demand, although it increases the share of high-quality buyers. The platform will be able to increase the total buyer demand, $D^b$, only by attracting more low-quality buyers, and one way of doing so is to increase the quality level of the basic (low-quality) service. More total buyer demand arising from increasing $q_l$ boosts the marginal revenue from the seller side due to the positive network effect on the platform. Thus, the optimal condition for $q_l$ is a function of the marginal revenue on the seller side.

Finally, we find that the additional markup from quality differentiation is increasing in the quality gap. Thus, if the platform sufficiently differentiates its product line with respect to quality, it can earn a higher markup. Proposition 6 summarizes this finding.

**Proposition 6.** *The additional markup charged to the high-quality buyers is increasing in the difference in quality, as shown in* $\frac{\partial\left(p_h^b/\epsilon_h^b\right)}{\partial(q_h-q_l)} > 0$.

Proposition 6 implies an interesting result—the platform earns more additional markup from high-quality buyers if either it provides much better service (e.g., one-day rather than two-day shipping for Prime members) or maintains the high-quality service at the same level while reducing the quality of the basic service. Section 5.2 discusses this point in depth.

For the last part of the equilibrium analysis, we examine the optimal conditions to derive further implications. We first examine whether the platform's decision variables are strategic complements or substitutes. We initially summarize the results for the one quality case.

**Proposition 7.** *The prices on the buyer side ($p^b$) and the seller side ($p^s$) are strategic substitutes. Whether the price variable (buyer or seller side) and quality variable are strategic substitutes or complements is ambiguous.*

From Proposition 7, reducing $p^b$ is the profit-maximizing response to increasing $p^s$, and vice versa. This means that the platform does not maximize its profits if it increases both prices. Unlike this case, the strategic relationship between price and quality variables is ambiguous. The price for the buyer and quality are strategic substitutes if $\frac{\partial^2 \Pi}{\partial p^b \partial q} = \frac{p^b}{Bq^2}[D^s f^b(\frac{p^b}{Bq}) - \frac{1}{Bq^2}(p^b + p^s - c(q))f'(\frac{p^b}{Bq})] + c'(q)\frac{1}{Bq} f^b(\frac{p^b}{Bq}) < 0$. Similarly, the price for the seller and quality are strategic substitute if $\frac{\partial^2 \Pi}{\partial p^s \partial q} = D^s \frac{p^b}{Bq^2} f^b(\frac{p^b}{Bq}) - c'(q)\frac{1}{S^2} f^s(\frac{p^s}{S}) < 0$. If $F^b$ and $F^s$ both have a uniform distribution, $\frac{\partial^2 \Pi}{\partial p^b \partial q}$ is always positive (because $f' = 0$), whereas $\frac{\partial^2 \Pi}{\partial p^s \partial q}$ is negative if $c'(q) > S(S - p^s)\frac{p^b}{Bq^2}$. This parametric example suggests that if the platform increases quality, it charges buyers a higher price but sellers a lower price (provided that the marginal cost of increasing quality is above a certain threshold).

Next, we derive results from the two qualities case. For simplicity, we assume that $F^b$ and $F^s$ both have a uniform distribution, which implies that the profit in the two qualities case is given as follows.

$$\Pi = \left(1 - \frac{p^s}{S}\right)\left[\left(p_h^b - q_h^2 + p^s\right)\left(\frac{p_l^b - p_h^b}{B\left(q_h - q_l\right)} + 1\right) + \left(1 - \frac{p_l^b}{Bq_l}\right)\left(p_l^b - q_l^2 + p^s\right)\right]. \tag{27}$$

By the first order conditions derived in 3.2, we find the following.

$$\begin{aligned}
\frac{\partial^2 \Pi}{\partial p_l^b \partial p_h^b} &= \frac{q_l}{2(q_h - q_l)} \\
\frac{\partial^2 \Pi}{\partial p_l^b \partial q_h} &= -\frac{q_l\left[p_h^b + q_h\left(q_h - 2q_l\right) + p^s\right]}{2\left(q_h - q_l\right)^2} \\
\frac{\partial^2 \Pi}{\partial p_l^b \partial q_l} &= \frac{1}{2}\left[B + \frac{q_h\left(p_h^b - q_h^2 + p^s\right)}{\left(q_h - q_l\right)^2} + 2q_l\right] \\
\frac{\partial^2 \Pi}{\partial p_l^b \partial p^s} &= \frac{q_h}{2(q_h - q_l)} - 1; \quad \frac{\partial^2 \Pi}{\partial p_h^b \partial q_h} = \frac{B}{2} + q_h \\
\frac{\partial^2 \Pi}{\partial p_h^b \partial q_l} &= -\frac{B}{2}; \quad \frac{\partial^2 \Pi}{\partial p_h^b \partial p^s} = -\frac{1}{2}.
\end{aligned} \tag{28}$$

From Equation (28), we identify whether two choice variables are strategic substitutes or complements for the platform. The following proposition summarizes the findings.

**Proposition 8.** *The price for high-quality access for buyers ($p_h^b$) is a strategic complement for the price for low-quality access for buyers ($p_l^b$) and high-quality service ($q_h$), whereas it is a strategic substitute for the price for sellers ($p^s$) and low-quality service ($q_l$). Whether the price for low-quality access for buyers is a strategic substitute or complement for quality measures and the price for the seller is ambiguous.*

Note that if the high-quality service is much better than the low-quality service,[10] $p_l^b$ is strategic substitute for $q_h$ but strategic complement for $p^s$. This implies that if the platform makes the high-quality service better ($q_h$ increases), the profit-maximizing response is to increase the price for high-quality access for buyers while decreasing that for low-quality access. In addition, the best response to an increase in the price for high-quality access for buyers is to decrease the price for sellers. However, if the platform increases the price for low-quality access for buyers, the optimal response is to increase the price for sellers provided that the quality difference is large enough.

We also conduct comparative statics to determine how changes in the exogenous parameters affect the equilibrium outcomes. In particular, we are interested in how $B$, which represents the basic benefit from the quality dimension for every buyer, affects consumer demand in the two qualities case. Proposition 9 summarizes the result.

**Proposition 9.** *Consumer demand for high-quality service always increases in the basic benefit from better quality ($B$). Whether consumer demand for low-quality service increases in $B$ is ambiguous.*

In other words, $\frac{\partial D_h^b}{\partial B}$ is always positive (where $D_h^b$ is given by Equation (14)), whereas $\frac{\partial D_l^b}{\partial B}$ is positive only if a certain condition is met (where $D_l^b$ is given by Equation (13)). Specifically, $\frac{\partial D_l^b}{\partial B}$ is positive if $\frac{p_l^b}{q_l} f^b\left(\frac{p_l^b}{Bq_l}\right) > \frac{p_h^b - p_l^b}{q_h - q_l} f^b\left(\frac{p_h^b - p_l^b}{B(q_h - q_l)}\right)$ and negative otherwise. That is, when the basic benefit from the quality dimension increases, it can reduce the buyers' demand for low-quality access if the price-quality ratio for the quality difference (between high- and low-quality) is greater than that for low-quality service. As in Equation (13), buyers in the middle range of willingness to pay (i.e., who are willing to pay for low-quality service but not for higher priced high-quality service) demand low-quality service access. As $B$ increases, we observe two

[10] We can guarantee that $\frac{\partial^2 \Pi}{\partial p_l^b \partial q_h} < 0$ and $\frac{\partial^2 \Pi}{\partial p_l^b \partial p^s} > 0$ if $q_h > 2q_l$.

simultaneous outcomes: (1) more buyers who were not in the market join the low-quality service (measured by $\frac{p_l^b}{q_l} f^b \left(\frac{p_l^b}{Bq_l}\right)$), and (2) more buyers who used to use the low-quality service switch to high-quality service (measured by $\frac{p_h^b - p_l^b}{q_h - q_l} f^b \left(\frac{p_h^b - p_l^b}{B(q_h - q_l)}\right)$). If the latter effect is larger than the first, a greater $B$ leads to fewer buyers for the low-quality service.

# 5 Discussion

Quality differentiation is one example of product differentiation, which makes buyers' demand less elastic. In other words, the platform can strategically use quality differentiation to maximize its profit. In Section 5, we discuss several business implications based on our theoretical predictions.

## 5.1 Quality differentiation and optimal pricing strategy

As in the model, if the platform provides different quality choices to buyers, it faces more inelastic demand from them, which allows the platform to charge a lower price to users on the other side, namely sellers. Specifically, the platform can extract a higher margin from buyers by providing multiple different qualities, which makes their demand inelastic. Given more inelastic demand from buyers, the platform finds it optimal to increase the number of sellers, as this increases the utility of buyers from interactions through the platform. The platform can increase the number of sellers by decreasing the price on the seller's side, which is one way of subsidizing sellers. There are several instances in which the platform might find it profitable to subsidize sellers by exploiting buyers' inelastic demand (arising from quality differentiation).

One such example is a credit card business that connects consumers and merchants. The model findings predict that if a credit card network provides two different cards, for example a premium card with a higher usage fee and a basic card with a lower usage fee, which makes consumers' demand inelastic (due to quality differentiation), the platform (the credit card network) is able to maximize its profit by charging a lower merchant discount fee (a price charged to merchants). In particular, if there is competitive pressure in attracting merchants, the platform can strategically exploit the quality differentiation on the consumer side to subsidize the merchant side, which helps it overcome intense competition on the merchant side. Suppose that the competition for merchants is so intense that it is difficult for the company to get merchants to accept its credit card. The platform wants to attract more merchants because widespread merchant acceptance is crucial for expanding its business. Here, if the platform charges much lower merchant discount fees, more merchants will be willing to accept its card. Our theory predicts that the platform will offer lower fees in equilibrium by differentiating on product quality on the consumer side, which leads to a higher merchant acceptance rate.

## 5.2 Quality differentiation and extra markup

The model predicts that the platform will earn more extra markup if it widens the quality gap. This suggests that the platform can adopt either of two strategies: it can improve the high-quality product while maintaining the low-quality product at the same level, or it can reduce the quality of the basic product while maintaining the high-quality product at the same level. Both strategies lead to more quality differentiation, which results in more extra markup being charged to the high-quality buyers per Proposition 6. Most incumbent platforms can use their pre-existing resources to develop a much higher quality product, thereby generating more quality differentiation (higher $q_h$). However, because entrants or small platforms lack substantial resources, they might not be able to make large investments to produce better quality products. What those platforms can do is to provide a very basic quality product at a lower price (or zero price) and a slightly higher quality product at a higher price (lower $q_l$). In other words, by lowering the basic product quality at an almost zero price (or free of charge), the platforms can enjoy more extra markup even without having very high product quality.

For example, there are two ways for Amazon to widen the quality gap. It could make its high-quality service more attractive by providing one-day shipping for Prime members. Alternatively, it could offer more differentiated services by maintaining the two-day shipping policy for Prime members while reducing some of the benefits for basic members, for example increasing the order minimum to qualify for free shipping from \$25 to \$30.

# 6 Conclusion

This paper analyzes a generalized version of quality differentiation by a monopolist in a multi-sided market. The main focus of this paper's analysis is the effect of buyers' side quality differentiation on the optimal pricing strategy for the platform. We first showed that quality differentiation on the buyer side will decrease the price charged to sellers. To understand the intuition behind this result, recall that the pricing structure for multi-sided firms depends on the relative elasticity of the two sides. Quality differentiation on the buyers' side can increase the surplus extracted from buyers, which means that the buyers' side becomes relatively inelastic. The platform can exploit this inelastic demand among buyers to extract more profit from that side while discounting the sellers' side by decreasing the price charged to them. This finding suggests how a platform can subsidize the more elastic side by introducing quality differentiation on the other side. We also found that quality differentiation, which leads to a lower price on the sellers' side, ultimately increases the platform's profit. Thus, the platform can strategically use quality differentiation to raise its profits. Another strategic variable that the platform can use is the extent to which the two qualities differ—the platform can earn extra markup from relatively high-valuation buyers by widening the quality gap. Given that the driving force of this greater markup is the quality gap, rather than how high the quality of the better quality product should be, we derived relevant business implications, especially for small platforms or entrants without substantial resources: if they are unable to make a higher quality product due to their limited resources, they can lower their basic product quality, which would lead to similar consequences in terms of the quality gap.

Overall, our findings suggest one plausible business strategy for the platform: how quality differentiation implemented by the platform can be used as an optimal business strategy. It would be interesting for future research to investigate how competition in the platform market alters our results. In this regard, if we were to consider an asymmetric setup in the competitive market structure faced by the platform, we could determine the extent to which such asymmetry affects the platform's optimal business strategy.

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# Appendix

Here, we include the proofs omitted from the main body.

**Proof of Proposition 1.** The proposition shows the platform offers a better price-quality ratio in the case of one quality offering. This basically implies

$$\underbrace{\frac{p^b}{q}}_{\text{Platform (two-sided)}} \leq \underbrace{\frac{p^b}{q}}_{\text{One-sided}} \tag{29}$$

We start by comparing the profit functions for two different cases. For simplicity, we normalize

$c^s$ to zero.

$$\begin{aligned}\Pi_{\text{one-sided}} &= [p^b - c(q)]D^b \equiv \Psi(p^b, q).\\ \Pi_{platform} &= [p^b + p^s - c(q)]D^b D^s \equiv \Psi(p^b, q)D^s + p^s D^b D^s.\end{aligned} \tag{30}$$

The one-sided monopolist's profit maximization problem is given in Equation (5). Let $\bar{x} = \left(\bar{p^b}, \bar{q}\right)$ be the optimal solution for the one-sided monopoly problem and $\widetilde{x} = \left(\widetilde{p^b}, \widetilde{q}, \widetilde{p^s}\right)$ for the two-sided platform. Given that the monopolist optimal value for $p^s \neq 0$,

$$\Psi(\widetilde{p^b}, \widetilde{q})\widetilde{D^s} + \widetilde{p^s}\widetilde{D^b}\widetilde{D^s} \geq \Psi(\bar{p^b}, \bar{q}) \tag{31}$$

As $(\bar{p^b}, \bar{q})$ is the optimal solution $\Pi_{\text{one-sided}}$ th;is implies that $\Psi(\bar{p^b}, \bar{q}) \geq \Psi(\widetilde{p^b}, \widetilde{q}) \geq \Psi(\widetilde{p^b}, \widetilde{q})\widetilde{D^s}$, so the only way Equation (31) holds is if

$$\begin{aligned} &\widetilde{D^s}\widetilde{p^s}(D^b)_{\text{at } \widetilde{x}} > \widetilde{D^s}\widetilde{p^s}(D^b)_{\text{at } \bar{x}}\\ \rightarrow &\underbrace{\frac{p_l^b}{q_l^b}}_{\text{platform}} < \underbrace{\frac{p_l^b}{q_l^b}}_{\text{one-sided}}. \end{aligned} \tag{32}$$

Hence proved. □

**Proof of Proposition 2.** We use proof by contradiction.

Let us suppose that $(p^s)_{1Q} < (p^s)_{2Q}$

**Step 1**: Some implications of the above assumptions are as follows:

$$(D^s)_{1Q} > (D^s)_{2Q} \tag{33}$$

$$\underbrace{(p^b + p^s - c(q^b))}_{\text{average profit for 1Q}} > \underbrace{(p_l^b + p^s - c(q_l^b)) + \frac{D_h^b}{D^b}((p_h^b - c(q_h)) - (p_l^b - c(q_l)))}_{\text{Avg profit for 2Q}} \tag{34}$$

Using equilibrium values for $p^s$ and that $\frac{p^s}{\epsilon^s}$ is decreasing in $p^s$, given Equation (34), the only way that the firm would provide two qualities is if

$$\underbrace{(D^sD^b)}_{2Q} > \underbrace{(D^sD^b)}_{1Q}$$
$$\rightarrow \frac{(D^b)_{2Q}}{(D^b)_{1Q}} > \frac{(D^s)_{1Q}}{(D^s)_{2Q}}$$

The above equation and Equation (33) imply

$$(D^b)_{2Q} > (D^b)_{1Q}$$
$$\rightarrow \underbrace{\frac{p^b}{q^b}}_{1Q} > \underbrace{\frac{p^b_l}{q_l}}_{2Q} \tag{35}$$

**Step 2**: Using $(p^s)_{1Q} < (p^s)_{2Q}$ in the equilibrium pricing structure conditions, we obtain

$$\underbrace{\frac{D^s}{(-D^s)'}}_{1Q} > \underbrace{\frac{D^s}{(-D^s)'}}_{2Q} \qquad \text{as this function is decreasing in price } (p^s)$$

$$\rightarrow \underbrace{\frac{D^b}{(-D^b)'_{p^b}}}_{1Q} > \underbrace{\frac{D^b}{(-D^b)'_{p^b_l}}}_{2Q} \tag{36}$$

$$\rightarrow \underbrace{\frac{q^b(1-F(\frac{p^b}{Bq^b}))}{f(\frac{p^b}{Bq^b})}}_{1Q} > \underbrace{\frac{q^b_l(1-F(\frac{p^b_l}{Bq^b_l}))}{f(\frac{p^b_l}{Bq^b_l})}}_{2Q} \tag{37}$$

$$\rightarrow \underbrace{\frac{q^b(1-F(\frac{c'(q^b)}{B}))}{f(\frac{c'(q^b)}{B})}}_{1Q} > \underbrace{\frac{q^b_l(1-F(\frac{c'(q^b_l)}{B})}{f(\frac{c'(q^b_l)}{B})}}_{2Q} \qquad \text{using equilibrium values of } q^b \text{ and } q_l \tag{38}$$

Now we apply the assumption that $\frac{\partial(\frac{c'(q)}{q})}{\partial q} > 0$ and $\frac{\partial(\frac{f^b(\theta)}{\theta})}{\partial\theta} \geq 0$. Using this assumption and the above equation, we obtain $q^b_l > q^b$. This can easily be shown by contradiction. Assume that $q^b > q_l^b$. Then, the cost assumption and Equation (38) imply:

$$\underbrace{\frac{c'(q^b)(1 - F(\frac{c'(q^b)}{B}))}{f(\frac{c'(q^b)}{B})}}_{1Q} > \underbrace{\frac{c(q_l^b)(1 - F(\frac{c'(q_l^b)}{B})}{f(\frac{c'(q_l^b)}{B})}}_{2Q}$$

$$\rightarrow \underbrace{\frac{c'(q^b)}{f(\frac{c'(q^b)}{B})}}_{1Q} > \underbrace{\frac{c(q_l^b)}{f(\frac{c'(q_l^b)}{B})}}_{2Q} \qquad \text{using } (D^b)_{2Q} > (D^b)_{1Q}$$

$$\rightarrow \underbrace{\frac{\theta}{f(\theta)}}_{1Q} > \underbrace{\frac{\theta}{f(\theta)}}_{2Q}$$

This contradicts assumption $\frac{\partial(\frac{f^b(\theta)}{\theta})}{\partial\theta} \geq 0$, and thus, $q^b > q_l^b$ cannot hold. Thus, we obtain $q_l^b > q^b$.

Using $q_l^b > q^b$ and Equation (37)[11], we obtain

$$\underbrace{\frac{p^b}{q^b}}_{1Q} < \underbrace{\frac{p_l^b}{q_l}}_{2Q}$$

which contradicts Equation (35). Hence, it is proven by contradiction. □

**Proof of Proposition 3.** Let $(\bar{p}^b, \bar{q}, \bar{p}^s)$ be the profit maximization variable for the single-quality case. We prove whether at this price and quality level, the platform wants to set a non-zero demand for high-quality products. The demand for high-quality products will be zero if:

$$\begin{aligned} &\frac{p_h^b - p_l^b}{B(q_h - q_l)} = 1 \\ &\rightarrow p_h^b - p_l^b = B(q_h - q_l) \end{aligned} \tag{39}$$

We choose $((p_h^b)^*, (q_h)^*)$ such that Equation (39) is satisfied and then determine whether the first order condition on $(p_h^b, q_h)$ shows that the platform will attempt to increase de-

[11] as $\frac{(1-F(\frac{p^b}{Bq^b}))}{f(\frac{p^b}{Bq^b})}$ is decreasing in $\frac{p^b}{q^b}$

mand for the high-quality product above zero. The first order condition with respect to $p_h^b$ at $(\bar{p}^b, \bar{q}, \bar{p}^s, (p_h^b)^*, (q_h)^*)$

$$\begin{aligned}
\Phi^{p_h^b} &= D^s \left\{ \{[(p_h^b)^* - c(q_h^*)] - [\bar{p}^b - c(\bar{q})]\}(D_h^b)'_{p_h^b} + D_h^b \right\} \\
&= D^s \left\{ \{[(p_h^b)^* - c(q_h^*)] - [\bar{p}^b - c(\bar{q})]\} (-f^b(1)) \right\} && \text{as } \frac{p_h^b - \bar{p}^b}{B(q_h - \bar{q})} = 1 \\
&= D^s \left\{ \{B(q_h^* - \bar{q}) - [c(q_h^*) - c(\bar{q})]\} (-f^b(1)) \right\} \\
&\leq 0 && \text{if } f^b(1) \neq 0 \text{ and } c'(\bar{q}) < B
\end{aligned} \tag{40}$$

In a similar manner, we can prove that at $q_h^*$, the first order condition is greater than zero. Thus, the platform will decrease $p_h^b$ and increase $q_h$ such that $D_h^b \neq 0$. Therefore, we see that the profit increases when offering two product qualities as long as $f^b(1) \neq 0$ and $c'(\bar{q}) < B$. $c'(\bar{q}) < B$ is true, as $c'(\bar{q}) < B$ implies that $\frac{\bar{p}}{B\bar{q}} < 1$, which holds for non-zero demand. □

**Proof of Proposition 4.** It is obvious to see from Equation (20).

**Proof of Proposition 5.** It is trivial to prove from checking Equation (22) and (23).

**Proof of Proposition 7.** The proof is in the paper.

**Proof of Proposition 8.** The proof is in the paper, from Equation (28).

**Proof of Proposition 9.** The proof is in the paper.