Quality Differentiation and Optimal Pricing Strategy in Multi-sided Markets*

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Abstract

This paper analyzes the generalized quality differentiation model in multi-sided markets with positive externalities, which leads to new insights into the optimal pricing structure of the firm. We find that quality differentiation for users on one side leads to a decrease in the price charged to users on the other side due to cross-subsidization, thereby affecting the pricing structure of multi-sided firms. In addition, quality differentiation affects the strategic relationships among the choice variables for the platform, so that the platform strategically uses quality differentiation to raise its profits.

Keywords: Multi-sided Market; Quality Differentiation; Platform Business Strategies
JEL Classification Numbers: D43; L11; L42

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1 Introduction

There has been an increasing shift toward multi-sided firms in many industries. The early pioneering models of multi-sided platforms were introduced by Armstrong (2006), Caillaud and Jullien (2003), Parker and Van Alstyne (2005), and Rochet and Tirole (2003). Importantly, many real-world organizations determine how close or how far they are from a multi-sided economic model based on changing industrial parameters. In this paper, we examine how platforms in multi-sided markets optimally use quality differentiation as a business strategy.

As many previous studies emphasize, it is important to establish how multi-sided platforms differ from typical one-sided firms and how such differences lead to new business implications. Many markets that have traditionally featured one-sided firms now feature more two-sided firms due to advanced technology; for example, the taxi industry only had one-sided firms before Uber appeared. Firms typically begin with a one-sided model and switch to a multi-sided model as they become more established. Doing so allows potential platforms to overcome the "chicken-and-egg" problem by first providing complementary goods themselves. For instance, Amazon started off as a pure retailer but has moved closer to a two-sided model over time by enabling third-party sellers to trade directly with consumers on its website. We can also find many other examples in which a firm faces a strategic choice of how many sides to pursue. For instance, in the personal computer market, Apple produces its own hardware, whereas Microsoft leaves this to independent manufacturers. As a result, Apple manages only a two-sided platform with consumers and software providers, while Microsoft manages a three-sided platform with consumers, software providers, and hardware providers. Indeed, there is an increasing shift from traditional business structures in which the firm focuses only on one side of users to platforms where the firm serves more than one side of users.

Given this shift toward multi-sided business, the main strategic choice that we analyze in this paper is the choice of the quality of interaction between users on two different sides. We focus on platforms that enable interaction between two sides, for example, buyers and sellers. The term "interaction" covers various traditional interactions, such as those observed in auction houses, and those on internet sites in person-to-business transactions, for example, Amazon. This also includes interactions or exchanges between application developers and application users on software platforms such as computers (e.g., Apple, Microsoft); mobile devices (e.g., iPhone, Samsung); and video games (e.g., Sony PlayStation, Xbox). These markets usually have more than one type of quality access on the buyers' side. For example, Amazon offers buyers two types of quality access; basic (low quality) access is free, while premium access (*Prime* membership, which is high quality) is the paid service. It is easy to see that the interactions or exchanges through the high-quality access have better quality—for example, *Prime* provides two-day shipping. Many ride-sharing services offer differentiated quality tiers for customers. For example, Uber provides riders with multiple types of quality services. Although each region may have different availability, there is basically a low-quality service at a cheaper price (e.g., *Uber X, Uber Pool*) as well as a high-quality service at a slightly higher price (e.g. *Uber XL, Uber Select, Uber Black*). For instance, whereas trips with *Uber Pool* may take longer time to complete because the car is shared by strangers, *Uber Select* or *Black* is a private car sharing service with a more luxurious car. Other ride-sharing services offer similar quality differentiation: Didi in China provides three quality tiers: *Express, Premier*, and *Luxe*. These are examples of quality differentiation by firms serving two sides. Ultimately, this paper provides business implications of quality differentiation in the multi-sided platform market by building a simple model of two-sided monopolists in a market with interactions between buyers and sellers. The firm chooses the price and the quality of interaction for the buyers' side.

The first main finding of the paper is that a firm provides higher quality per dollar to buyers as it serves more sides. That is, when a firm expands its business from one-sided service (serving buyers/riders only) to multi-sided service as a platform (serving both buyers/riders and sellers/drivers), it provides better quality to buyers. Intuitively, if a firm offers a better-quality price menu to buyers, it attracts more buyers. When it serves the sellers' side at the same time, more demand from the buyers' side makes sellers on the other side earn more revenue. The platform can extract those additional revenues on the sellers' side, which incentivizes the platform to offer a better-quality price ratio to buyers.

We also find that the quality differentiation on the buyers' side decreases the price charged on the sellers' side compared to the price charged by a single-quality, two-sided firm. The intuition behind this finding is related to product differentiation and demand elasticity. If the platform provides multiple quality options for buyers, it means that it provides more differentiated products, which makes buyers more inelastic to price changes because the platform can extract extra surplus from high-quality buyers without deterring buyer participation. As the buyers' side becomes more inelastic, it creates an incentive for the platform to extract a higher rent from the buyers' side and subsidize the sellers' side by lowering the price faced by sellers.

By providing more quality choices, even if the quality differentiation is small, the platform is able to obtain more profits. Given that buyers are heterogeneous in their valuation of product quality, there is an incentive for the platform to offer different levels of product quality at different prices to extract more rents from buyers. When adopting quality differentiation, the platform needs to charge a lower price for low-quality buyers. However, it is able to earn higher markup from the high-valuation group of buyers (who will buy a high-quality product) if the quality gap is widened. Given that the number of high-quality buyers is sufficiently large, the platform's ultimate profit is greater with quality differentiation. Based on the model predictions, we discuss some business implications for how quality differentiation on one side of the market helps the platform raise its profit.

Related literature There is a broad literature on the corresponding problem of a monopolistic firm seeking to maximize profits by offering quality-differentiated products in one-sided standard markets. The seminal papers are Spence (1977), Mussa and Rosen (1978), Maskin and Riley (1984), and M. Itoh (1983). We generalize this problem by varying the number of sides served by the firm.

This paper is also related to the literature on pricing structure in markets with multi-sided firms (e.g., Rochet and Tirole, 2003; Armstrong, 2006b; Reisinger, 2010). A major difference in this paper is that we consider a three-way interaction among prices for buyers and sellers and quality choice for the optimal pricing structure in multi-sided firms. In addition to being broadly related to the literature on pricing in multi-sided markets, papers on skewed pricing in multi-sided markets are closely related to our paper in terms of our theoretical implications. Suarez and Cusumano (2008) discuss the platform's subsidy pricing strategy to attract greater user adoption, although they do not set up an economic model to confirm this strategy. Bolt and Tieman (2008), Schmalensee (2011), and Dou and Wu (2018) study skewed pricing strategies in two-sided markets, i.e., the subsidy and money sides. However, those papers do not consider forms of product differentiation, such as the quality differentiation examined in our paper, as a means of skewing prices. Additionally, Sridhar et al. (2011) focus on cross-market network effects in two-sided markets, as we do in this study: however, their main focus is on how the optimal marketing investment allocation is affected by cross-market effects.

Regarding markets with multi-sided firms, few papers have focused on firms choosing the quality of interaction on their platform. These papers study markets with negative network externalities and do not endogenize the quality choice. Crampes and Haritchabalet (2009) examine the choice of offering a pay ads regime and no pay ads packages. Peitz and Valletti (2004) compare the advertising intensity when media operators offer free services and when the subscription price is positive. Viecens (2006) is an exception because she studies a setup with endogenous quality differentiation on two-sided platforms. However, the quality differentiation in her model takes a different form from ours in that she focuses on the quality provided by users on one side and not on the quality provided by the platform itself. Therefore, her results do not provide any implications for the platform's dynamic pricing structure, such as subsidizing one side at the expense of the other. Another quality-related aspect explored in the context of two-sided markets is the case in which users care about the quality of the other users with whom they interact, which is relevant for matching markets such as dating sites. Jeon et al. (2016) examine this problem in a platform setting. Renato and Pavan (2016) consider this problem in a matching setup. Hagiu (2012) studies a model in which users value the average quality of other users. The setup in these papers, however, is different from that in ours in that we focus on the quality of interaction and not on the quality of users.

Our analyses are also related to the literature on product differentiation in multi-sided markets, in that quality differentiation is one form of product differentiation. Smet and Cayseele (2010) focus on product differentiation in platform markets, which still differs from our paper in that they do not account for its consequences for optimal pricing strategies.

The literature has not focused on endogenizing the choice of network quality in multi-sided markets with positive externalities. Therefore, our results on how multi-sided markets are combined with quality differentiation on a platform provide new insights into the related business.

2 Model Setup

We model the interaction between buyers and sellers. Economic value is created through the interaction between these two sides. We consider the case of a single firm providing a platform for interactions between buyers and sellers. For instance, Amazon acts as a transaction-based platform for interactions between sellers and buyers. In addition to charging an access fee to use the platform, the firm can also control the quality of the interaction. Figure 1 displays the structure of the two-sided firm with quality differentiation on the buyers' side using Amazon as an example. The two types of access quality offered on the buyers' side are Amazon *Basic* and *Prime*—Amazon *Prime* offers higher quality access, as it provides premium services including free two-day shipping.

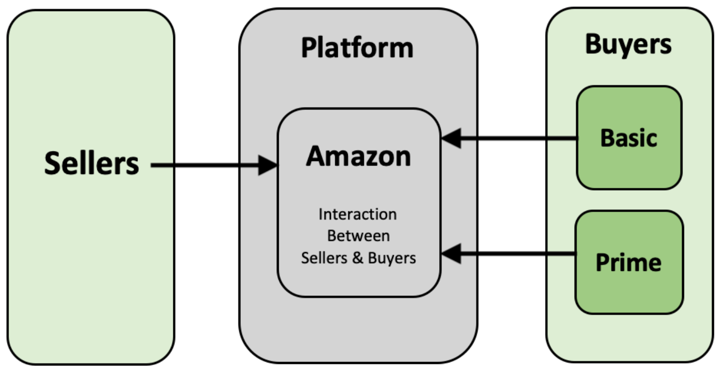

Figure 1: Two-sided firm with quality differentiation

Buyer's side Specifically, we model a two-sided monopolist firm that price discriminates on the buyers' side by offering two different types of access quality: $q_k \in \{q_l, q_h\}$, where k denotes the quality provision, either $l$ow or $h$igh. In this model, the users' gain from the platform is derived through interaction between users on two sides: buyers and sellers. Both buyers and sellers obtain utility from interacting with each other. The quality variable controls the gain from each such interaction. We assume that both buyers and sellers are heterogeneous with respect to the per-interaction or usage benefit. The usage or per-interaction benefits are $b_i^b(q_k)$ for the buyers' side (for individual buyer i) and b_j^s for the sellers' side (for individual seller j), where the superscript b (s) denotes buyers (sellers). For simplicity, we assume that $b_i^b(q_k) = B\alpha_i^b q_k$, where B represents the basic benefit for every buyer. This benefit is dependent on the quality of interaction; thus, the monopolist can control the benefit by choosing the quality of platform access q_k on the buyers' side. The term α_i^b denotes the heterogeneity among buyers; it follows a distribution function F^b with support on $[0, 1]$ and density f^b. Additionally, each buyer pays a price for using the platform, which is denoted p_k^b, as a usage or per-transaction fee. For a buyer i, the utility function is given by the following:

$$
\begin{aligned}
&U^b_{ik} = [b^b_i(q_k) - p^b_k]N^s, \quad \text{where } k \in \{l, h\} \\
&\Leftrightarrow U^b_{ik} = (B\alpha^b_i q_k - p^b_k)N^s.
\end{aligned} \tag{1}
$$

The buyers' utility is the net benefit from each interaction with the other side, i.e., $(B\alpha^b_i q_k - p^b_k)$, multiplied by the number of interactions, which is denoted N^s.[1] From the utility specification, it can be shown that the buyer with the highest benefit is that with $\alpha^b_i = 1$, i.e., $B\alpha^b_i = B$.

We assume that the access fee p^b_k is charged per-transaction for simplicity. Indeed, certain kinds of platforms charge for premium service based on usage: for Uber, a basic service (*Uber X*) is cheaper than a premium service (*Uber Select*) for the same trip. We can see that the per-usage fee for the basic service (low quality) is zero (i.e., $p^b_l = 0$ in this example) whereas that for the premium service (high quality) is the difference between two riding costs (i.e., $p^b_h > 0$). Nevertheless, it is worth noting that many platforms charge a one-time fixed access fee: for instance, Amazon *Prime* costs nothing for *Basic* but \$119 per year (or \$12.99 per month) for *Prime* service. Note that the qualitative results hold under the model with fixed fees, as shown in the Appendix A.2. Such one-time fixed fee structure can also be considered as a variation of a per-transaction fee by considering the fixed fee as continuous: if Amazon *Prime* members make hundred transactions per year on average, the average per-transaction cost is \$1.19 (i.e., $p^b_l = 0$ and $p^b_h = \$1.19$).

Seller's side The sellers' side is not affected by quality; therefore, there is no quality component. We assume that $b^s_j = S\alpha^s_j$, where α^s_j represents seller j's heterogeneity with respect to the per-usage benefit, which is also distributed by a distribution function F^s with support on [0,1] and density f^s. For seller j, the utility function is given by the following:

$$
U^s_j = (b^s_j - p^s)N^b = (S\alpha^s_j - p^s)N^b. \tag{2}
$$

The sellers' utility is the net benefit from each interaction with the other side, i.e., $S\alpha^s_j - p^s$, multiplied by the number of interactions, which is denoted N^b.[2] All sellers are charged p^s per

[1] We assume here that every buyer interacts with every seller and that every seller interacts with every buyer. We can easily extend this to a model where the number of interactions is a function of the total number of sellers, such as $g(N^s)$. The results are robust to such extensions.

[2] In this model, we assume that the sellers' side is affected only by the total number of buyers, not the type of buyer with which a seller interacts. Thus, the total number of interactions for each seller is N^b, and the total number of interactions by sellers is not affected by the type of buyer with which they interact.

interaction, so the price discrimination is only on the buyers' side—for example, an individual seller on Amazon pays \$0.99 for each sale. The total number of interactions is $N^b N^s$.[3]

The platform Next, the cost of the two-sided monopoly firm depends on the quality provided. The total cost of a transaction is given by $c(q_l) \geq 0$ for a transaction between a low-quality buyer and a seller and $c(q_h) \geq 0$ for a transaction between a high-quality buyer and a seller. In the Amazon example, such cost differentiation captures that two-day shipping for *Prime* members is costlier than standard shipping for *Basic* members. We normalize the cost for sellers $c^s = 0$. The cost function is assumed to be increasing and convex in quality ($c'(q) > 0$, $c''(q) > 0$). We analyze the nontrivial case in which $q_h > q_l$.

The demand on the buyers' and sellers' sides is represented by D^b and D^s, respectively. In equilibrium, demand will be equal to the number of participants on each side, which means $D_k^b = N_k^b$ and $D^s = N^s$, where $k \in \{l, h\}$. Given the equilibrium demands, we turn to the monopolistic platform's problem. The monopoly platform's problem can be written as follows:

- If one type of quality is offered:

$$\max_{p^s, p^b, q} \Pi = [p^b + p^s - c(q)] D^b D^s. \tag{3}$$

- If two types of quality are offered:

$$\max_{p^s_{2Q}, p^b_l, p^b_h, q_l, q_h} \Pi = [p_l^b + p_{2Q}^s - c(q_l)] D_l^b D^s + [p_h^b + p_{2Q}^s - c(q_h)] D_h^b D^s, \tag{4}$$

where p^s_{2Q} denotes the price for sellers when two quality levels are offered to buyers.

Note that we endogenize the platform's decision whether to provide quality differentiation. As we will show, the platform prefers quality differentiation with high and low quality levels to buyers because it is more profitable than one-quality provision. Still, we analyze the case of one-quality provision in Section 2.1 for a benchmark.

Throughout the paper, we make the following assumptions.

Assumption 1. *The cost is increasing and convex in quality:* $c'(q) > 0$, $c''(q) > 0$.

This suggests that it becomes increasingly costly to provide higher quality service, which is a standard assumption.

[3] Note that the qualitative results still hold under a generalized setup with any function of $N^b N^s$.

Assumption 2. *The distribution functions for buyers and sellers are increasing in type:* $(f^b)' \geq 0$ *and* $(f^s)' \geq 0$*, which implies that* $f^b(1) > 0$ *and the inverse hazard rate is non-increasing in user type:* $\frac{\partial \frac{1-F^b(\theta)}{f^b(\theta)}}{\partial \theta} \leq 0$ *and* $\frac{\partial \frac{1-F^s(\theta)}{f^s(\theta)}}{\partial \theta} \leq 0$*.*

Assumption 2 implies that we impose the standard monotone hazard rate condition on F. Additionally, it guarantees that as users attribute more value to the per-usage benefit, there are more platform users than nonusers.

Assumption 3. $\frac{\partial U^b(\alpha_i^b, p^b, q)}{\partial p^b} < 0$, $\frac{\partial U^b(\alpha_i^b, p^b, q)}{\partial q} > 0$, $\frac{\partial U^s(\alpha_i^s, p^s)}{\partial p^s} < 0$.

Assumption 4. *The single-crossing property holds:* $\frac{\partial^2 U^b(\alpha_i, q(\alpha_i))}{\partial \alpha_i \partial q} > 0$*.*

This means that we can always distinguish high-type buyers from low-type buyers based on their α_i.

Timing of the game The timing of the game is set as follows.

0. Before the main game begins, the platform is assumed to provide one quality q to buyers at p^b, which respectively are the basic (low) quality level and the price. The price for sellers is set at p^s.

1. The platform decides whether to provide two levels of quality to buyers. If engaging in quality differentiation, it determines both quality levels (basic low q_l and high q_h) and the associated price levels for buyers (p_l^b and p_h^b). It also optimally sets the price for seller p_{2Q}^s.

2. Buyers and sellers make participation decisions.

When the platform introduces quality differentiation in Stage 1, it is likely to be subject to a certain price commitment to buyers. Without quality differentiation, buyers pay p^b for the basic quality service measured by q. When the platform changes its quality provision with two qualities, q_l and q_h, it may want to keep faith with the existing buyers who used to pay p^b for quality level q by guaranteeing them the same price level. To reflect this scenario, we restrict our attention to the case in which the platform fixes the price for low-quality service,

p_l^b, at the price level absent quality differentiation, p^b.[4] Thus, in the main analysis, we take p_l^b in the two-quality provision case as given by p^b. For the quality choice, we allow the platform to offer a different basic low quality ($q \neq q_l$) even at the same price ($p^b = p_l^b$) to examine how the platform uses quality differentiation flexibly as an alternative to a fixed price under price commitment. This assumption permits us to analyze the platform's business strategies regarding quality differentiation in reality. For instance, after Amazon introduced a *Prime* membership tier that costs \$99 with free two-day shipping service, it raised the minimum order amount eligible for free shipping for non-Prime *Basic* members from \$35 to \$49 in 2016. Although Amazon maintained its zero price for these members with basic tier quality service even after introducing *Prime* membership, i.e., $p^b = p_l^b = \$0$, the provided basic quality level, as reflected in the increase in its free shipping minimum, was reduced, i.e., $q > q_l$.[5] In Section 4.2, we also look at the case in which the platform commits to providing the same quality level at the same price (i.e., $q = q_l$ and $p^b = p_l^b$) after introducing a high-quality service (q_h at the price of p_h^b).

The solution concept we use for this game is the Perfect Bayesian Nash Equilibrium (PBE) for multiperiod games with observed action, which can be solved by backward induction. PBE consists of a sequentially rational strategy profile for all players and a set of consistent beliefs about buyers' and sellers' valuations with respect to benefits from using the platform.

2.1 Model with one-quality case

As a benchmark, we begin by considering a model that omits the practice of quality differentiation and then modify the model to allow for differentiation in Section 2.2. The two-sided monopolist offers a single quality of access in this case. Buyers have the following utility:

$$U_i^b = (B\alpha_i^b q - p^b)N^s, \tag{5}$$

[4]Note that we further assume that the platform's pricing decision in the prestage (Stage 0) is not affected by its ability to engage in quality differentiation in the later stage. In this sense, we can say that the platform is myopic insofar as it does not set p^b in Stage 0 (under one-quality provision) by taking into account any changes in the choice variables that are set in Stage 1 under quality differentiation such as p_h^b and p_{2Q}^s.

[5]Refer to https://www.wired.com/2016/02/now-amazons-free-shipping-will-cost-you-at-least-49/. Additionally, note that Amazon has continued adjusting its free shipping minimum for basic members. In 2017, it reduced the minimum order amount for free shipping back to \$35 from \$49, followed by another reduction to \$25. Such changes reflect the changes in the low-quality level (from q to q_l) with two-quality provision in our model.

where $q \in [0,1]$ denotes the quality index. The seller has the following utility:

$$U_j^s = (S\alpha_j^s - p^s)N^b. \tag{6}$$

We first analyze the users' side (buyers and sellers) to identify the equilibrium demand. The equilibrium demand functions are derived from the participation constraint:

$$\begin{aligned} D^b &= Prob\left(U_i^b \geq 0\right) \Leftrightarrow D^b = 1 - F^b\left(\frac{p^b}{Bq}\right). \\ D^s &= Prob\left(U_j^s \geq 0\right) \Leftrightarrow D^s = 1 - F^s\left(\frac{p^s}{S}\right). \end{aligned} \tag{7}$$

The monopoly problem can be written as follows:

$$\max_{p^s, p^b, q} \Pi = [p^b + p^s - c(q)]D^b D^s, \tag{8}$$

where we have normalized the marginal cost on the seller's side to zero, i.e., $c^s = 0$. We can solve for the optimal solutions for the buyers' and sellers' sides as follows:

$$\frac{p^b - c(q) + p^s}{p^b} = \frac{1}{\varepsilon^b}; \quad \frac{p^s - c(q) + p^b}{p^s} = \frac{1}{\varepsilon^s}. \tag{9}$$

Equation (9) is similar to the standard Lerner pricing formula, which states that the markup on price is equal to the inverse of the price elasticity of demand. The price elasticity of demand for buyers (or sellers) is represented by ε^b (or ε^s). The added term in the case of a two-sided firm is the extra opportunity cost of increasing the price on the buyers' (or sellers') side, which is the marginal loss on the sellers' (or buyers') side, equal to p^s (or $p^b - c(q)$).

Finally, the optimal condition for quality is given as follows:

$$c'(q) = \frac{[p^b - c(q) + p^s]}{D^b}\frac{\partial D^b}{\partial q}. \tag{10}$$

The optimal quality equates the marginal cost to the marginal revenue of increasing quality. The marginal revenue is the change in buyers' demand,[6] which is represented by $\frac{\partial D^b}{\partial q}$ times the per-transaction profit. From the conditions given above, the equilibrium price-quality structure is given by Theorem 1.

[6]In the optimal quality equation, the term for the number of sellers (D^s) is canceled out, as it is present on both sides of the equation.

Theorem 1. *The equilibrium price and quality variables are given by the following equation:*

$$\frac{qc'(q)}{\nu^b} = \frac{p^b}{\varepsilon^b} = \frac{p^s}{\varepsilon^s}, \tag{11}$$

where ν^b is the quality elasticity of demand on the buyers' side, and ε^b or ε^s is the price elasticity of the buyer or seller side, respectively.

Equation (11) is similar to the equilibrium of the two-sided market with a quality decision by the platform, as in Rochet and Tirole (2003). However, our equilibrium condition still differs from that in Rochet and Tirole (2003) insofar as we focus on a three-way trade-off that includes the quality variable as well instead of a trade-off between prices on the two sides. By placing the main focus on price variables only, Rochet and Tirole (2003) find that the price on the buyers' side would be lower if sellers are more price elastic because the platform wants to balance the sizes of the two sides. Theorem 1 further says that the higher quality the platform offers, the higher price it charges to buyers, while such upward pressure on prices diminishes as buyers become more quality sensitive.

Given that many platforms consider their quality of service as another important decision variable, our finding broadens the necessary considerations for the platform: it needs to set optimal prices not only by considering the price structures in terms of the ratio of price elasticities, as pointed out in Rochet and Tirole (2003), but also by taking quality elasticity into account, as we additionally find. For instance, if buyers on Amazon become more elastic to quality increases, Amazon is able to increase the buyers' side demand substantially even with a minimal quality improvement. Accordingly, if Amazon changes its shipping policy for *Prime* members to three-day, instead of two-day, and lowers the fee charged to *Prime* members (i.e., p^b is lower than \$119 per year, which is currently set for two-day shipping benefits), this makes Amazon better off because more *Basic* members will sign up for *Prime* membership: the demand increasing effect arising from a small quality improvement outweighs the price decreasing effect due to high-quality elasticity.

Allowing the platform to consider how high or low quality a service to offer, or how the quality tiers it provides are differentiated, creates more opportunities for optimal business strategies, insofar as quality choice offers another level of flexibility in terms of profit maximization for the platform. Indeed, as we will show, the platform's optimal pricing decisions depend on its quality choice.

2.2 Model with two qualities case

As mentioned earlier, we focus on the case in which the platform makes a price commitment for the basic low-quality service, i.e., $p^b = p_l^b$. For now, we allow the platform to vary its basic low-quality level when a high-quality service is introduced. Thus, the choice variables for the platform include q_l, q_h, p_h^b, and p_{2Q}^s, whereas p_l^b is given by the equilibrium p^b, which is determined in Section 2.1.

We start by analyzing the users' side (buyers and sellers) to identify the equilibrium demand. First, the buyers have two choices for accessing the platform. They can join the platform through either low-quality access or high-quality access. Given the two types of quality, high and low, the number of participants joining with low-quality access is determined by the number of buyers who satisfy the following two conditions:

1. (IR constraint) The buyers' utility from low-quality access is greater than zero: $Pr(U_l^b \geq 0)$.
2. (IC constraint) Buyers for whom the utility derived from low-quality access exceeds that from high-quality access: $Pr(U_l^b \geq U_h^b)$.

The two conditions jointly determine the proportion of low-type buyers.

$$D_l^b = \Pr\left(\frac{p_h^b - p^b}{B(q_h - q_l)} \geq \alpha_i^b \geq \frac{p^b}{Bq_l}\right) = F^b\left(\frac{p_h^b - p^b}{B(q_h - q_l)}\right) - F^b\left(\frac{p^b}{Bq_l}\right), \tag{12}$$

where $D_l^b \equiv D^b(p_h^b, p^b, q_h, q_l)$. Similarly, the number of participants joining the high-quality service is given by the number of buyers who satisfy the following two conditions:

1. (IR constraint) The buyers' utility from the high-quality good is greater than zero: $Pr(U_h^b \geq 0)$.
2. (IC constraint) Buyers for whom the utility derived from the high-quality good exceeds that from the low-quality good: $Pr(U_h^b \geq U_l^b)$.

The IR condition is satisfied when the IC constraint of the high type and IR constraint of the low type holds.[7] Thus, the proportion of high-type buyers is given by:

$$D_h^b = \Pr\left(\alpha_i^b \geq \frac{p_h^b - p^b}{B(q_h - q_l)}\right) = 1 - F^b\left(\frac{p_h^b - p^b}{B(q_h - q_l)}\right), \tag{13}$$

[7] We maintain the standard single-crossing condition, which implies that higher types have greater willingness to pay (WTP) for quality at any price or that consumers may be ordered by their type.

where $D_h^b \equiv D^b(p_h^b, p^b, q_h, q_l)$. Given the utility function for buyers, the total number of buyers joining the platform is given by the following:

$$\begin{aligned} D^b &= \Pr(U_l^b \geq 0) = \Pr(b_l^b \geq p^b) \\ \Leftrightarrow D^b &= \Pr(B\alpha_i^b q_l \geq p^b) = \Pr(\alpha_i^b \geq \frac{p^b}{Bq_l}) = 1 - F^b\left(\frac{p^b}{Bq_l}\right), \end{aligned} \tag{14}$$

where $D^b \equiv D(p^b, q_l)$. Equation (14) shows us how the number of participants on the buyers' side depends only on the price and quality of the low-quality good. Although there are network externalities in the total utility derived from the platform or the gross transaction utility, the per-unit transaction demand is not dependent on the participation rate on the other side.[8] This is because the participation constraint (IR constraint) for the high-quality buyers is slack. This means that the participation of low-quality buyers guarantees the participation of high-type buyers. In other words, the buyers on the margin of joining the platform are low-quality buyers.

Next, the total number of sellers who join the platform is given by the following:

$$D^s = \Pr(U^s \geq 0) = 1 - F^s\left(\frac{p_{2Q}^s}{S}\right), \tag{15}$$

where $D^s \equiv D^s(p_{2Q}^s)$. Given the total number of buyers and sellers, the equilibrium level of participation is the following:

$$\begin{aligned} &D^b = D(p^b, q_l) = 1 - F^b\left(\frac{p^b}{Bq_l}\right); \quad D^s = D^s(p_{2Q}^s) = 1 - F^s\left(\frac{p_{2Q}^s}{S}\right). \\ &D_l^b = F^b\left(\frac{p_h^b - p^b}{B(q_h - q_l)}\right) - F^b\left(\frac{p^b}{Bq_l}\right); \quad D_h^b = D^b(p_h^b, p^b, q_h, q_l) = 1 - F^b\left(\frac{p_h^b - p^b}{B(q_h - q_l)}\right). \end{aligned} \tag{16}$$

Given quality differentiation, the monopoly problem can be written as follows:

$$\max_{p_{2Q}^s, p_h^b, q_l, q_h} \Pi = [p^b + p_{2Q}^s - c(q_l)]D_l^b D^s + [p_h^b + p_{2Q}^s - c(q_h)]D_h^b D^s. \tag{17}$$

The following is the breakdown of the equilibrium prices and quality for buyers and sellers.[9]

2.2.1 Price of high-quality access on the buyers' side

The price of high-quality access on the buyers' side can be obtained as follows:

[8] This setup has one restriction that we need to impose, which is that the proportion of low-type buyers has to be nonnegative: $D_l^b \geq 0$.

[9] The details are in the Appendix.

$$p_h^b = p^b + \underbrace{[c(q_h) - c(q_l)] + \frac{p_h^b}{\varepsilon_h^b}}_{\text{additional cost plus extra market power}}, \tag{18}$$

where the price elasticity of demand for high-quality access is represented by ε_h^b. The optimal price for high-quality access is equal to the price for low-quality access and the additional cost, i.e., $c(q_h) - c(q_l)$, plus an additional markup, i.e., $\frac{p_h^b}{\varepsilon_h^b}$.

2.2.2 Price for sellers

We now turn our attention to the price for sellers.

$$p^b + p_{2Q}^s - c(q_l) = \frac{p_{2Q}^s}{\varepsilon^s} - \frac{p_h^b}{\varepsilon_h^b}\frac{D_h^b}{D^b}. \tag{19}$$

Again, Equation (19) is similar to the standard Lerner pricing formula, which states that the markup on price is equal to the inverse of the price elasticity of demand. The added term in the case of a two-sided firm is the extra opportunity cost of increasing the price on the sellers' side, which is the marginal loss on the buyers' side, and is equal to the average per-interaction profit on the buyers' side.[10]

2.2.3 Low-quality service for buyers

Given prices, the monopolistic platform solves the profit maximization problem separately for the low- and high-quality services for buyers. First, for the low-quality service, the first order condition can be derived as follows:

$$p^b + p_{2Q}^s - c(q_l) = \frac{q_l}{\nu^b}\frac{D_l^b c'(q_l) + D_h^b c'(q_h)}{D^b}, \tag{20}$$

where ν^b denotes the quality elasticity of demand for low-quality access. Note that the equilibrium conditions for the quality elasticity with respect to the lower quality are the same as those for the one-quality provision case: that is why we have ν^b not ν_l^b. The cost of low-quality service equates the marginal cost to the marginal revenue of increasing quality.

[10]Note that we have normalized the cost on the seller's side to zero, so $c^s = 0$.

2.2.4 High-quality service for buyers

Here, the first order condition for the high-quality service for buyers is given by the following:

$$c'(q_h)\frac{q_h}{\nu_h^b} = \frac{p_h^b}{\varepsilon_h^b}, \tag{21}$$

where ν_h^b denotes the quality elasticity of demand for high-quality access. The marginal cost $c'(q_h)$ should be equal to the marginal revenue, which is the product of increased high-quality buyers, i.e., $\frac{\nu_h^b}{q_h}$, and the extra markup generated from the increase in high-quality buyers, i.e., $\frac{p_h^b}{\varepsilon_h^b}$.

3 Equilibrium Results

We derive several important implications from the model by showing how quality differentiation determined by the multi-sided platform affects its optimal use of the interaction between two sides, such as cross-subsidization, in the following subsections.

Before we proceed, we first show that the platform has an incentive to engage in quality differentiation in the first place: we find that quality differentiation leads to greater profit for the platform, as in Proposition 1. The detailed proof is in the Appendix.

Proposition 1. *The platform strictly prefers to price discriminate by quality on the buyers' side.*

Proposition 1 implies that a platform that provides only one type of quality is able to obtain more profit if it slightly differentiates product quality. Even a minor quality improvement with a small price increase can increase the platform's profit as long as it continues to provide differentiated products, such as low- and high-quality products. Thus, quality differentiation permits the platform to earn more profit by implementing premium service in addition to basic service, which not only expands the size of the total buyer market but also extracts more rents from the relatively high-quality type of buyers.

In the following subsections, we focus on the multi-sided platform case with the provision of two quality levels to investigate how quality differentiation on the buyer's side affects the price for sellers, the price for buyers, and the interaction between the two sides.

3.1 The effect of quality differentiation on the sellers' side

We first find that quality differentiation can reduce the other side's, i.e., sellers' side, price level under some conditions. First, quality differentiation leads to a fall in p^s, i.e., $p^s > p^s_{2Q}$. That is, introducing quality differentiation decreases the price for sellers. The Appendix contains a proof of this result.

Proposition 2. *Quality differentiation on the buyers side decreases the price charged on the sellers side relative to the price charged by a platform that offers a single quality.*

The effect of offering low- and high-quality access to buyers makes the buyers more price inelastic because the platform can extract additional surplus from high-quality buyers without deterring participation. Now, recall that for multi-sided firms, the optimal pricing scheme is to subsidize the more elastic side of the market and extract rents from the other more inelastic side. As the buyers side becomes more inelastic, it creates an incentive for the platform to extract higher rent from the buyers side and subsidize the sellers side by lowering the price charged to sellers.

Similar to the single-quality case, the platform faces a trade-off between whether to charge a higher price on the buyers or sellers side in cases with two quality types. After quality differentiation, the trade-off features an additional markup benefit on the buyers side, namely, the extra margin from a high-quality buyer, as shown in the equilibrium condition below:

$$\begin{aligned} &\textit{one-quality case: } \frac{p^s}{\varepsilon^s} = \frac{p^b}{\varepsilon^b(p^b, q)} \\ &\textit{two-quality case: } \frac{p^s_{2Q}}{\varepsilon^s} = \frac{p^b}{\varepsilon^b(p^b, q_l)} + \underbrace{\frac{D^b_h}{D^b}\frac{p^b_h}{\varepsilon^b_h}}_{\text{extra markup from quality differentiation}} \end{aligned} \tag{22}$$

Thus, the monopolist becomes more efficient in extracting rent from the buyers side. Given this result, the platform has a greater incentive to increase seller demand by lowering the price on the sellers side.

3.2 The effect of quality differentiation on the buyers' side

Here, we find that the additional markup from quality differentiation is increasing in the quality gap. Thus, if the platform sufficiently differentiates its product line with respect to quality, it can earn a higher markup. Proposition 3 summarizes this finding.

Proposition 3. *The platform's incentive to raise the fee charged to high-quality buyers is increasing in the difference in quality, as shown in* $\frac{\partial\left(p_h^b/\varepsilon_h^b\right)}{\partial(q_h-q_l)} > 0$.

Proposition 3 implies an interesting result—the platform is more likely to charge a higher fee to high-quality buyers if it either provides much better service (e.g., one-day rather than two-day shipping for Amazon *Prime* members) or maintains the high-quality service at the same level while reducing the quality of the basic service (e.g., increasing the minimum order to qualify for free shipping for Amazon *Basic* members). Section 5.2 discusses this point in depth.

3.3 The effect of quality differentiation on the interaction between the two sides

Here, we examine the strategic relationships between the platform's choice variables to derive further implications.

Proposition 4. *The price on the seller's side (p^s) is a strategic complement to the low-quality level (q_l) on the buyer's side. Furthermore, the high-quality level and the associated price on the buyer's side (p_h^b and q_h) have no direct effect on p^s*

Proposition 4 first states that the seller's side choice variable strategically interacts only with low-quality service-related variables, and not with those related to high-quality service. This finding further specifies how the seller's side equilibrium is strategically related to the buyer's side equilibrium (in low-quality service). Per Proposition 2, we find that quality differentiation lowers the price charged on the sellers' side. As the platform decreases the price charged to sellers, it strategically reduces its basic low-quality level. For example, if Uber charges lower driver pay rate (i.e., p^s falls) after introducing differentiated quality tiers (e.g., *Uber Pool vs. Uber Select*), its optimal choice for the basic low-quality service, which is *Uber Pool*, is to lower its quality by increasing waiting time (i.e., q_l decreases) even at the same price p^b.

Combined with Proposition 3, given that a deterioration in the basic low-quality level raises the price charged to high-quality buyers, Proposition 4 illuminates how the platform, Uber, cross-subsidizes under quality differentiation by optimally extracting additional surplus from high-quality riders. Following the basic quality deterioration after introducing the high-quality tier (i.e., $q_l < q$), the platform extracts more revenue from high-quality buyers through a higher p_h^b. Although worse basic quality under two-quality provision shrinks the total buyers' demand, the platform still finds it profitable per Proposition 1. Thus, upon quality differentiation, the platform subsidizes the sellers' side by lowering p^s by extracting higher rent from high-quality buyers (i.e., higher p_h^b) at the expense of low-quality buyers (i.e., lower q_l).

Next, we conduct further comparative statistics to determine how changes in the exogenous parameters affect the equilibrium outcomes. In particular, we are interested in how B, which represents the basic benefit from the quality dimension for every buyer, affects consumer demand in the case of two qualities. Proposition 5 summarizes the result.

Proposition 5. *Consumer demand for the high-quality service always increases in the basic benefit from better quality (B). Whether consumer demand for low-quality service increases in B is ambiguous.*

In other words, $\frac{\partial D_h^b}{\partial B}$ is always positive (where D_h^b is given by Equation (13)), whereas $\frac{\partial D_l^b}{\partial B}$ is positive only if a certain condition is met (where D_l^b is given by Equation (12)). Specifically, $\frac{\partial D_l^b}{\partial B}$ is positive if $\frac{p^b}{q_l} f^b\left(\frac{p^b}{Bq_l}\right) > \frac{p_h^b - p^b}{q_h - q_l} f^b\left(\frac{p_h^b - p^b}{B(q_h - q_l)}\right)$ and negative otherwise. That is, when the basic benefit from the quality dimension increases, it can reduce the buyer's demand for low-quality access if the price-quality ratio for the quality difference (between high and low quality) is greater than that for low-quality service. As in Equation (12), buyers in the middle range of willingness to pay (i.e., those who are willing to pay for low-quality service but not for higher priced high-quality service) demand low-quality service access. As B increases, we observe two simultaneous outcomes: (i) more buyers who were not in the market join the low-quality service (measured by $\frac{p^b}{q_l} f^b\left(\frac{p^b}{Bq_l}\right)$), and (ii) more buyers who previously used the low-quality service switch to the high-quality service (measured by $\frac{p_h^b - p^b}{q_h - q_l} f^b\left(\frac{p_h^b - p^b}{B(q_h - q_l)}\right)$). If the latter effect is larger than the former, a greater B leads to fewer buyers for the low-quality service. According to Proposition 5, we can see that considering quality differentiation for the platform affects its strategic choices for the price and demand structure. If the platform overlooks the dynamic relationship between quality choice and optimal pricing, it could overlook potential better business strategies using the three-way interaction between quality, buyers' side, and sellers' side.

4 Discussion

4.1 The combined effects of openness and quality differentiation

Thus far, we considered the platform to be a multi-sided firm and have not investigated how the effects of quality differentiation for the multi-sided platform are different from those for a one-sided firm. By comparing the effects of quality differentiation on a one-sided firm to those on a two- or multi-sided firm, we can see how serving more sides as the platform, which we call openness in the paper, is affected differently by quality differentiation through exploiting the interaction between different sides.

If the monopolist serves only one side of the market, say the buyers' side, its profit function is given by the following:

$$\max_{p^b,q} \Pi_{\text{one-sided monopolist}} = [p^b - c(q)]D^b, \tag{23}$$

in the case of one quality.

We find that the monopolistic firm provides higher quality to buyers for each dollar that they pay when it serves more sides of the market. As the monopolist opens more sides to serve, say from a one-sided firm serving buyers only to a two-sided firm as a platform serving both buyers and sellers, such openness increases the quality per dollar offered to buyers. Mathematically, the quality per dollar is denoted as $\frac{q}{p^b}$. This finding is summarized in Proposition 6.

Proposition 6. *As a firm serves more sides of the market, it provides higher quality per dollar offered to buyers:* $\frac{q}{p^b}^{\textit{One-sided firm}} \leq \frac{q}{p^b}^{\textit{Two-sided firm}}$.

Intuitively, the monopolist serving multiple sides has more incentive to offer a better-quality price ratio to buyers because it now obtains more profit from the seller's side. If the two-sided firm offers a better-quality price menu to buyers, it attracts more of them. When it serves the sellers' side at the same time, more demand from the buyers' side means that the sellers on the other side earn more revenue. The platform can extract the additional revenues on the sellers' side, which incentivizes it to offer a better-quality price ratio to buyers. In other words, opening the platforms from serving only one side of the user base to serving both sides increases the quality per dollar, which ultimately attracts more buyers than a one-sided platform.

Moreover, both traditional one-sided firm and two-sided platforms offer differentiated quality tiers for buyers. When those two types of firms engage in quality differentiation on the buyers' side, the logic in Proposition 6 can be also interpreted as follows: servicing both sides makes the platform more willing to offer a better deal to low-type members by providing high quality per dollar in the basic level than a one-sided firm. For example, the average quality of services provided by taxi companies, as traditional one-sided firms, is known to be lower than that provided by ride-sharing platforms, such as Uber and Lyft. As Liu et al. (2019) show, taxi drivers are more likely to detour with non-local customers, which results in longer travel time. Such empirical evidence supports our finding that the platform's basic quality provision is better than that of a one-sided firm. The reason the platform provides a higher quality for basic service is that it is able to exploit such quality improvement in one side to encourage more participation from the other side, thereby raising its profits. As in the ride-sharing platform example, when such a platform provides higher basic quality to riders, thereby attracting more riders, it ultimately creates a stronger incentive for drivers to join, thereby allowing it to extract more rent from drivers. The underlying incentives for platforms to engage in higher service provision than a one-sided firm arise from this cross-subsidization motive. This finding is summarized in Corollary 1.

Corollary 1. *The platform in a two- or multi-sided market is more likely to offer a higher quality basic service or product than a one-sided firm.*

4.2 Price and quality commitment by the platform

Thus far, when introducing high-quality service, we have assumed that the platform keeps charging the basic low-quality service at p^b, as in the case with one-quality access, while it varies the low-quality level itself. Therefore, the platform lowers the low-quality level below the basic single quality it previously offered (i.e., $q_l < q$), thereby lowering the per dollar quality level ($\frac{q_l}{p^b} < \frac{q}{p^b}$). This corresponds to the case in which after Amazon introduced *Prime* membership as a high-quality service offering free two-day shipping, it lowered the basic service quality by increasing the order minimum to qualify for free shipping from \$25 to \$30 (i.e., $q > q_l$) but maintained free basic membership (i.e., $p^b = p^b_l = 0$). However, the platform may need to maintain both price and quality for the low-tier service at the same levels as in the case of one-quality provision: $p^b = p^b_l$ and $q = q_l$. If the platform has less flexibility when engaging in quality differentiation by fixing both price and quality level for the low-tier service at the same level as in the single-quality provision, we find that the platform raises the price for sellers after quality differentiation. This finding is summarized in Proposition 7.

Proposition 7. *If the platform introduces quality differentiation with high-quality service while maintaining the basic quality level as well as its price at the same level as in one-quality provision, quality differentiation on the buyers' side raises the price for sellers.*

Per Propositions 2 and 4, cross-subsidization between the sellers' and buyers' sides partly arises from the link between the optimal q_l and p^s. However, if the platform faces a constraint in adjusting the basic low-quality level, such that q_l is fixed at q, the level with one-quality provision, in addition to p_l^b being fixed at p^b, it loses its link for cross-subsidization; therefore, p^s increases after quality differentiation.

5 Managerial Implications

Quality differentiation is one example of product differentiation, which makes buyers' demand less elastic. In other words, the platform can strategically use quality differentiation to maximize its profit. In Section 5, we discuss several business implications based on our theoretical predictions.

5.1 Quality differentiation and optimal pricing strategy

As in the model, if the platform provides different quality choices to buyers, it faces more inelastic demand from them, which allows the platform to charge a lower price to users on the other side, namely sellers. Specifically, the platform can extract a higher margin from buyers by providing multiple different qualities, which makes their demand inelastic. Given more inelastic demand from buyers, the platform finds it optimal to increase the number of sellers, as this increases the utility of buyers from interactions through the platform. The platform can increase the number of sellers by decreasing the price on the sellers' side, which is one way of subsidizing sellers.

There are several instances in which the platform might find it profitable to subsidize sellers by exploiting buyers' inelastic demand (arising from quality differentiation). One such example is a ride-sharing service that connects riders and drivers such as Uber. The model findings predict that if Uber provides two or multiple different quality services, for example, a premium service with a higher usage fee and a basic service with a lower usage fee, which makes consumers' demand inelastic (due to quality differentiation), the platform (Uber) will be able to maximize its profit by charging a lower driver pay rate (the price charged to drivers). In particular, if there is competitive pressure in attracting drivers, the platform can strategically exploit the quality differentiation on the riders' side to subsidize the drivers' side, which helps it overcome intense competition on the drivers' side. Suppose that the competition for drivers is so intense, after Lyft and other competitors start operating, that it is difficult for the company to obtain sufficient drivers during peak periods. The platform wants to attract more drivers because widespread availability of drivers is crucial for expanding its business. Here, if Uber charges much lower fees for drivers, more drivers will be willing to work for the platform. Our theory predicts that the platform will offer lower fees in equilibrium by differentiating on product quality on the riders' side, which leads to a higher driver participation rate.

5.2 Quality differentiation and extra markup

The model predicts that the platform will earn more extra markup if it widens the quality gap. This suggests that the platform can adopt either of two strategies: it can improve the high-quality product while maintaining the low-quality product at the same level, or it can reduce the quality of the basic product while maintaining the high-quality product at the same level. Both strategies lead to more quality differentiation, which results in more extra markup being charged to the high-quality buyers per Proposition 3.

Most incumbent platforms can use their pre-existing resources to develop a much higher quality product, thereby generating more quality differentiation (higher q_h). However, because entrants or small platforms lack substantial resources, they might not be able to make large investments to produce better-quality products. What those platforms can do is to provide a very basic quality product at a lower price (or zero price) and a slightly higher quality product at a higher price. In other words, by lowering the basic product quality, for which it charges an almost zero price (or makes it free of charge), a platform can enjoy greater extra markup even without very high product quality. For example, there are two ways for Amazon to widen the quality gap. It could make its high-quality service more attractive by providing one-day shipping for *Prime* members. Alternatively, it could offer more differentiated services by maintaining the two-day shipping policy for *Prime* members while reducing some of the benefits for basic members, for example, increasing the order minimum to qualify for free shipping from $25 to $30.

6 Conclusion

This paper analyzes a generalized version of quality differentiation by a monopolist in a multi-sided market. The main focus of this paper's analysis is the effect of buyers' side quality differentiation on the optimal pricing strategy for the platform. We first showed that quality differentiation on the buyers' side will decrease the price charged to sellers. This finding suggests how a platform can subsidize the more elastic side by introducing quality differentiation on the other side.

We also found that quality differentiation, which leads to a lower price on the sellers' side, ultimately increases the platform's profit. Thus, the platform can strategically use quality differentiation to raise its profits. Another strategic variable that the platform can use is the extent to which the two qualities differ—the platform can earn extra markup from relatively high-valuation buyers by widening the quality gap. Given that the driving force of this greater markup is the quality gap, rather than how high the quality of the better-quality product should be, we derive relevant business implications, especially for small platforms or entrants without substantial resources: if they are unable to make a higher quality product due to their limited resources, they can lower their basic product quality, which would lead to similar consequences in terms of the quality gap.

Overall, our findings suggest one plausible business strategy for the platform: how quality differentiation implemented by the platform can be used as an optimal business strategy. It would be interesting for future research to investigate how competition in the platform market alters our results. In this regard, a model with an asymmetric setup in the competitive market structure faced by the platform could determine the extent to which such asymmetry affects the platform's optimal business strategy.

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Appendix

A Further Discussion

A.1 Details of the equilibrium price and quality equations

The monopoly problem as in Equation (4) can be rewritten as follows:

$$\max_{p^s_{2Q},p^b_h,q_l,q_h} \Pi = [p^b + p^s_{2Q} - c(q_l)]D^bD^s + [(p^b_h - c(q_h)) - (p^b - c(q_l))]D^b_hD^s. \tag{24}$$

Using Equation (24), the first order conditions for choice variables can be simplified as follows.

- **Price for high-quality access on the buyers' side:** Equation (18) can be obtained by the following steps.

$$\{[p^b_h - c(q_h)] - [p^b - c(q_l)]\}(D^b_h)'_{p^b_h}D^s + D^b_hD^s = 0.$$
$$\Leftrightarrow \quad \{[p^b_h - c(q_h)] - [p^b - c(q_l)]\} = \frac{D^b_h}{(-D^b_h)'_{p^b_h}} = \frac{p^b_h}{\varepsilon^b_h},$$

where $(D^b_h)'_{p^b_h}$ means that $\frac{\partial D^b_h}{\partial p^b_h}$.

- **Price for sellers:** Equation (19) can be obtained by the following steps.

$$\{[p^b + p^s_{2Q} - c(q_l)]D^b + \{[p^b_h - c(q_h)] - [p^b - c(q_l)]\}D^b_h\}(D^s)'_{p^s_{2Q}} + D^bD^s = 0.$$
$$\Leftrightarrow \quad p^b + p^s_{2Q} - c(q_l) = \frac{D^s}{(-D^s)'_{p^s_{2Q}}} - \{[p^b_h - c(q_h)] - [p^b - c(q_l)]\}\frac{D^b_h}{D^b}.$$

Using Equation (18), $p^b + p^s_{2Q} - c(q_l) = \frac{D^s}{(-D^s)'_{p^s_{2Q}}} - \left[\frac{p^b_h}{\varepsilon^b_h}\right]\frac{D^b_h}{D^b} = \frac{p^s_{2Q}}{\varepsilon^s} - \left[\frac{p^b_h}{\varepsilon^b_h}\right]\frac{D^b_h}{D^b},$

where $(D^s)'_{p^s_{2Q}}$ means that $\frac{\partial D^s}{\partial p^s_{2Q}}$.

- **Low-quality service for buyers:** Equation (20) can be obtained by the following steps.

$[\{(D^b - D^b_h)c'(q_l) + [p^b + p^s_{2Q} - c(q_l)](D^b)'_{q_l} + [(p^b_h - c(q_h)) - (p^b - c(q_l))](D^b_h)'_{q_l}\}D^s = 0.$

Using Equations (18) and (21), $-D^b_lc'(q_l) + [p^b + p^s_{2Q} - c(q_l)](D^b)'_{q_l} + \left[-\frac{D^b_h}{(D^b_h)'_{p^b_h}}\right](D^b_h)'_{q_l}$

Finally, using $(D^b_h)'_{q_l} = -(D^b_h)'_{q_h}$, $-(D^b_l)c'(q_l) + [p^b + p^s_{2Q} - c(q_l)](D^b)'_{q_l} - c'(q_h)D^b_h = 0,$

which is simplified as, $[p^b + p^s_{2Q} - c(q_l)] = \frac{D^b}{(D^b)'_{q_l}}\frac{D^b_lc'(q_l) + D^b_hc'(q_h)}{D^b} = \frac{q_l}{\nu^b}\frac{D^b_lc'(q_l) + D^b_hc'(q_h)}{D^b},$

where $-\frac{D^b_h}{(D^b_h)'_{p^b_h}} = c'(q_h)\frac{D^b_h}{(D^b_h)'_{q_h}}$ and $(D^b_k)'_{q_k}$ means that $\frac{\partial D^b_k}{\partial q_k}$ and $(D^b_k)'_{q_j}$ means that $\frac{\partial D^b_k}{\partial q_j}$, where $j \neq k$.

- **High-quality service for buyers:** Equation (21) can be obtained by the following steps.

  $\{[p^b_h - c(q_h)] - [p^b - c(q_l)]\}(D^b_h)'_{q_h} - D^b_h c'(q_h) = 0.$

  Using Equation (18), $c'(q_h)\dfrac{D^b_h}{(D^b_h)'_{q_h}} = \dfrac{p^b_h}{\varepsilon^b_h}$, which can be simplified as, $c'(q_h)\dfrac{q_h}{\nu^b_h} = \dfrac{p^b_h}{\varepsilon^b_h}$.

A.2 Proof of equivalence of the main model with the fixed fee case

Suppose that on one of the sides, the platform also charges a fixed fee. For example, Amazon charges a fixed fee, i.e., *Prime* membership, on the buyers' side. The key to the equivalence is that adding a fixed fee does not change the model as long as there are no fixed benefits for the users. This case fits the Amazon example, as even though Amazon charges a fixed fee, the buyers only benefit through transactions and receive no non-transaction benefits from Amazon.

Without loss of generality, let us assume that the buyers' side is also charged a fixed fee, which implies that buyers have the following utility:

$$U^b_i = (B\alpha^b_i q - p^b)N^s - P^b, \tag{25}$$

where P_b is the fixed fee charged to the buyers. The seller has the following utility:

$$U^s_j = (S\alpha^s_j - p^s)N^b. \tag{26}$$

We first analyze the users' side (buyers and sellers) to identify the equilibrium demand. The equilibrium demand functions are derived from the participation constraint:

$$\begin{aligned} D^b &= Prob\left(U^b_i \geq 0\right) \Leftrightarrow D^b = 1 - F^b\left(\frac{p^b + \frac{P^b}{N^s}}{Bq}\right). \\ D^s &= Prob\left(U^s_j \geq 0\right) \Leftrightarrow D^s = 1 - F^s\left(\frac{p^s}{S}\right). \end{aligned} \tag{27}$$

The monopoly problem can be written as follows:

$$\max_{p^s, p^b, q} \Pi = [p^b + p^s - c(q)]D^b D^s + P^b D^b. \tag{28}$$

Now, we modify the above case to show the equivalence. Let $p^b_{new} = p^b + \frac{P^b}{N^s}$ be the per transaction fee on the buyers' side and let the fixed fee be zero. Buyers have the following utility:

$$
\begin{aligned}
U_i^b &= (B\alpha_i^b q - p^b)N^s - P^b. \\
&= (B\alpha_i^b q - p^b - \frac{P^b}{N^s})N^s. \quad = (B\alpha_i^b q - p^b_{new})N^s.
\end{aligned} \tag{29}
$$

Thus, this shows the utility is the same in the case of (i) usage fee p^b as the fixed fee, and (ii) usage fee of p^b_{new}. The seller has the following utility:

$$
U_j^s = (S\alpha_j^s - p^s)N^b. \tag{30}
$$

We first analyze the users' side (buyers and sellers) to identify the equilibrium demand. The equilibrium demand functions are derived from the participation constraint:

$$
\begin{aligned}
D^b &= Prob\left(U_i^b \geq 0\right) \Leftrightarrow D^b = 1 - F^b\left(\frac{p^b_{new}}{Bq}\right). \\
D^s &= Prob\left(U_j^s \geq 0\right) \Leftrightarrow D^s = 1 - F^s\left(\frac{p^s}{S}\right).
\end{aligned} \tag{31}
$$

The monopoly problem can be written as follows:

$$
\max_{p^s, p^b, q} \Pi = [p^b + p^s - c(q)]D^b D^s + P^b D^b \quad = [p^b + p^s - c(q) + \frac{P^b}{D^s}]D^b D^s = [p^b_{new} + p^s - c(q)]D^b D^s. \tag{32}
$$

Thus, as shown above, the profit function for the platform and the users utilities are the same in these two cases. □

B Proofs of Propositions

Proof of Proposition 1. Let $(\bar{p}^b, \bar{q}, \bar{p}^s)$ be the profit maximization variable for the single-quality case. We prove whether the platform wants to set a nonzero demand for high-quality products at this price and quality level. The demand for high-quality products will be zero if

$$
\frac{p_h^b - p^b}{B(q_h - q_l)} = 1. \;\Leftrightarrow\; p_h^b - p^b = B(q_h - q_l). \tag{33}
$$

We choose $((p_h^b)^*, (q_h)^*)$ such that Equation (33) is satisfied and then determine whether the first order condition on (p_h^b, q_h) shows that the platform will attempt to increase demand for the high-quality product above zero. The first order condition with respect to p_h^b

at $(\bar{p}^b, \bar{q}, \bar{p}^s, (p_h^b)^*, (q_h)^*)$ is given as follows:

$$
\begin{aligned}
\Phi^{p_h^b} &= D^s \left\{ \{[(p_h^b)^* - c(q_h^*)] - [\bar{p}^b - c(\bar{q})]\}(D_h^b)'_{p_h^b} + D_h^b \right\}. \\
&= D^s \left\{ \{[(p_h^b)^* - c(q_h^*)] - [\bar{p}^b - c(\bar{q})]\} (-f^b(1)) \right\} \quad \text{as } \frac{p_h^b - \bar{p}^b}{B(q_h - \bar{q})} = 1. \\
&= D^s \left\{ \{B(q_h^* - \bar{q}) - [c(q_h^*) - c(\bar{q})]\} (-f^b(1)) \right\}. \\
&\leq 0 \quad \text{if } f^b(1) \neq 0 \text{ and } c'(\bar{q}) < B.
\end{aligned} \tag{34}
$$

In a similar manner, we can prove that at q_h^*, the first order condition is greater than zero. Thus, the platform will decrease p_h^b and increase q_h such that $D_h^b \neq 0$. Therefore, we see that the profit increases when offering two product qualities as long as $f^b(1) \neq 0$ and $c'(\bar{q}) < B$. $c'(\bar{q}) < B$ is true, as $c'(\bar{q}) < B$ implies that $\frac{\bar{p}}{B\bar{q}} < 1$, which holds for nonzero demand. □

Proof of Proposition 2. Comparing the equilibrium condition for the two-quality case to that for the one-quality case, we have the following:

$$
\textit{one-quality case:} \qquad \frac{p^s}{\varepsilon^s} = \frac{qc'(q)}{\nu^b}. \tag{35}
$$

$$
\textit{two-quality case:} \qquad \frac{p_{2Q}^s}{\varepsilon^s} = \frac{q_l}{\nu^b} \frac{D_l^b c'(q_l) + D_h^b c'(q_h)}{D^b} + \underbrace{\frac{D_h^b}{D^b} \frac{p_h^b}{\varepsilon_h^b}}_{\text{extra margin from high-quality buyers}}. \tag{36}
$$

We prove this by contradiction. Assume that $p^s < p_{2Q}^s$. By Assumption 2 for both the buyers' and sellers' distributions, this yields:

$$
\begin{aligned}
& \frac{p^s}{\varepsilon^s} > \frac{p_{2Q}^s}{\varepsilon^s} \\
\Leftrightarrow \quad & \frac{qc'(q)}{\nu^b} > \frac{q_l}{\nu^b} \frac{D_l^b c'(q_l) + D_h^b c'(q_h)}{D^b} + \frac{D_h^b}{D^b} \frac{p_h^b}{\varepsilon_h^b}. \\
\Leftrightarrow \quad & \frac{qc'(q)}{\nu^b} > \frac{q_l c'(q_l)}{\nu^b} \quad \text{as } q_l < q_h
\end{aligned} \tag{37}
$$

Note that the function $\frac{qc'(q)}{\nu^b}$ is increasing in q; thus,

$$
q > q_l. \tag{38}
$$

Let us revisit the first order condition for the low-quality level in the two-quality case (as in Equation (19)).

$$
c(q_l) - \frac{p_h^b}{\varepsilon_h^b} \frac{D_h^b}{D^b} = p^b + p_{2Q}^s - \frac{p_{2Q}^s}{\varepsilon^s}. \tag{39}
$$

Similarly, for the one-quality case, we rewrite the equilibrium condition as follows (using Equation (10)):

$$c(q) = p^b + p^s - \frac{p^s}{\varepsilon^s}. \tag{40}$$

Using Equations (39) and (40) and the assumption $p^s < p^s_{2Q}$, we obtain

$$c(q) < c(q_l) - \frac{p^b_h}{\varepsilon^b_h}\frac{D^b_h}{D^b} \quad \Leftrightarrow \quad c(q) < c(q_l)$$

As this function is increasing in the quality level, we obtain $q < q_l$, which contradicts Equation (38). Thus, p^s decreases to p^s_{2Q} as long as $\frac{\partial(\frac{(1-F(\theta))}{f(\theta)})}{\partial(\theta)} < 0$ holds. □

Proof of Proposition 3. First, $\frac{p^b_h}{\varepsilon^b_h}$ is given as follows.

$$\frac{p^b_h}{\varepsilon^b_h} = \frac{D^b_h}{(-D^b_h)'_{p^b_h}} = (q_h - q_l)\frac{1 - F(\frac{p_h - p_l}{q_h - q_l})}{f(\frac{p_h - p_l}{q_h - q_l})}. \tag{41}$$

By partially differentiating Equation (41) with respect to $q_h - q_l$, we have the following.

$$\frac{\partial \frac{D^b_h}{(-D^b_h)'_{p^b_h}}}{\partial(q_h - q_l)} = \frac{1 - F(\frac{p_h - p_l}{q_h - q_l})}{f(\frac{p_h - p_l}{q_h - q_l})} + (q_h - q_l)\frac{\partial\left(\frac{1 - F(\frac{p_h - p_l}{q_h - q_l})}{f(\frac{p_h - p_l}{q_h - q_l})}\right)}{\partial(q_h - q_l)} > 0,$$

which completes the proof. □

Proof of Proposition 4. The monopoly problem as in Equation (4) can be rewritten as follows:

$$\max_{p^s_{2Q}, p^b_h, q_l, q_h} \Pi = [p^b + p^s_{2Q} - c(q_l)]D^b D^s + [(p^b_h - c(q_h)) - (p^b - c(q_l))]D^b_h D^s. \tag{42}$$

Differentiating with respect to p^s is given as:

$$\begin{aligned} \frac{\partial \Pi}{\partial p^s_{2Q}} =& \{[p^b + p^s_{2Q} - c(q_l)]D^b + \{[p^b_h - c(q_h)] - [p^b - c(q_l)]\}D^b_h\}(D^s)'_{p^s_{2Q}} + D^b D^s \\ =& \{[p^b + p^s_{2Q} - c(q_l)]D^b D^s + \{[p^b_h - c(q_h)] - [p^b - c(q_l)]\}D^b_h D^s\}\frac{(D^s)'_{p^s_{2Q}}}{D^s} + D^b D^s \\ =& \Pi + D^b D^s. \end{aligned} \tag{43}$$

Now differentiating with respect to q_l, we obtain $\frac{\partial \Pi}{\partial q_l} + D^s \frac{\partial D^b}{\partial q_l}$. Since q_l maximizes Π, this leads to $\frac{\partial^2 \Pi}{\partial p^s_{2Q} q_l} = 0 + D^s \frac{\partial D^b}{\partial q_l} > 0$. Thus, p^s_{2Q} and q_l are strategic complements.

Using Equation (43), we can also show that p^b_h and q_h are neither strategic substitutes nor complements to the price on the sellers' side. □

Proof of Proposition 5. The proof is in the paper.

Proof of Proposition 6. This proof shows that the platform offers a better price-quality ratio in the case of one quality offering. This implies

$$\underbrace{\frac{p^b}{q}}_{\text{Platform (two-sided)}} \leq \underbrace{\frac{p^b}{q}}_{\text{One-sided}}. \tag{44}$$

We start by comparing the profit functions for two different cases. For simplicity, we normalize c^s to zero.

$$\begin{aligned}\Pi_{\text{one-sided}} &= [p^b - c(q)]D^b \equiv \Psi(p^b, q). \\ \Pi_{\text{platform}} &= [p^b + p^s - c(q)]D^b D^s \equiv \Psi(p^b, q)D^s + p^s D^b D^s.\end{aligned} \tag{45}$$

The one-sided monopolist's profit maximization problem is given in Equation (23). Let the optimal solution be $\bar{x} = \left(\bar{p^b}, \bar{q}\right)$ for the one-sided monopoly problem and $\widetilde{x} = \left(\widetilde{p^b}, \widetilde{q}, \widetilde{p^s}\right)$ for the two-sided platform. Given that the monopolist's optimal value for $p^s \neq 0$,

$$\Psi(\widetilde{p^b}, \widetilde{q})\widetilde{D^s} + \widetilde{p^s}\widetilde{D^b}\widetilde{D^s} \geq \Psi(\bar{p^b}, \bar{q}). \tag{46}$$

As $(\bar{p^b}, \bar{q})$ is the optimal solution $\Pi_{\text{one-sided}}$, this implies that $\Psi(\bar{p^b}, \bar{q}) \geq \Psi(\widetilde{p^b}, \widetilde{q}) \geq \Psi(\widetilde{p^b}, \widetilde{q})\widetilde{D^s}$, so Equation (46) holds only if

$$\begin{aligned}&\widetilde{D^s}\widetilde{p^s}(D^b)_{\text{at } \widetilde{x}} > \widetilde{D^s}\widetilde{p^s}(D^b)_{\text{at } \bar{x}}. \\ \Leftrightarrow\ & \underbrace{\frac{p^b}{q}}_{\text{platform}} < \underbrace{\frac{p^b}{q}}_{\text{one-sided}}.\end{aligned} \tag{47}$$

Hence, the proof is complete. □

Proof of Corollary 1. It is guaranteed by Proposition 6.

Proof of Proposition 7. Assuming that the platform is not able to change the price and quality of the existing lower quality access, the lower quality access will be the same as the one-quality access, which means that

$$p^b = p_l^b; \quad q = q_l. \tag{48}$$

Let us re-evaluate the first order condition for the seller's price in two-quality provision as follows.

$$\phi_{2Q}(p^s) \equiv p^b + p^s - c(q) - \frac{p^s}{\varepsilon^s} + \left[\frac{p_h^b}{\varepsilon_h^b}\right]\frac{D_h^b}{D^b}. \tag{49}$$

Additionally, we denote the first order condition with respect to p^s in one-quality provision as $\phi_{1Q} \equiv p^b + p^s - c(q) - \frac{p^s}{\varepsilon^s}$ and the optimal prices for sellers in both cases are given by p^s and p_{2Q}^s. Given that $\phi_{2Q}(p_{2Q}^s) = 0$, the following holds: $\phi_{2Q}(p^s) = \cancelto{0}{\phi_{1Q}(p^s)} + \left[\frac{p_h^b}{\varepsilon_h^b}\right]\frac{D_h^b}{D^b} > 0 = \phi_{2Q}(p_{2Q}^s)$. As ϕ_{2Q} is concave at p^s, we have $p^s < p_{2Q}^s$. □