

Move Order in Hotelling Models of Platform Markets*

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Abstract

We study a Hotelling framework in which customers pay a platform to enter a market in which they make a purchase from one of two competing sellers on opposite ends of a Hotelling line. We solve for a full-coverage equilibrium outcome when either the platform or the sellers have the first move. Whereas the outcome in a game where sellers move first can resemble that in the canonical game with no platform, when the platform moves first, the unique outcome is a limiting case of the outcome under partial coverage in which the platform uses its first-mover advantage to reduce competing sellers' equilibrium prices.

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1 Introduction

The Hotelling framework remains a methodological mainstay in theoretical platform market research. The framework has been used to model competition between rival platforms (e.g., Rochet and Tirole, 2003; Armstrong, 2006; Hagiu, 2009) as well as competition between rival sellers who rely on a platform to get their product to consumers.¹ In the latter characterization, consumers must compensate the platform in order to access a market of interest and also pay the Hotelling transportation cost associated with their preferred good (Choi and Kim, 2010; Guo et al., 2010; Cheng et al., 2011) as well as possibly a price charged by the seller of that good (Baranes, 2014; Fudickar, 2015; Gautier and Somogyi, 2020).

We are specifically interested in a scenario in which consumers must compensate, both, the platform and sellers to procure a good. This typifies the market for subscription Internet content, in which consumers pay an Internet service provider along with a content provider like Disney Plus or HBO Max. It could also exemplify competition between product suppliers at a club warehouse like Costco Wholesale, where consumers pay a fixed membership fee and separately for products from competing suppliers on which the warehouse makes little to no margin.

To model effective competition between sellers, researchers generally assume a parametrization that ensures full coverage over the Hotelling line, leading the sellers to compete for the marginal consumer. Outside a platform setting this leads to a well known, straightforward solution method whereby the marginal consumer determines sellers' demand functions and sellers simultaneously set prices to maximize profit. A seemingly natural extension of this to a platform setting is for a platform to have the first move where it sets a non-discriminatory market entrance fee that leaves the marginal consumer indifferent between exiting or remaining in the market. As we show, in the extension, seller prices are bounded by the constraint that the indifferent consumer's utility equals zero *conditional on the equilibrium value of the market entrance fee*. Ignoring the equilibrium value of the market entrance fee can lead to overstating seller prices and understating the platform fee.²

To determine the market entrance fee under full coverage, we instead first find what it would be under partial coverage and then find the limit of the partial coverage equilibrium outcome as it approaches full coverage. In the unique full coverage outcome, the platform is able to use its first-mover advantage to induce rival sellers to set prices to half those that would result in a standard

¹We note that following the maximum differentiation result of D'Aspremont et al. (1979), correcting a deficiency in Hotelling (1929), it has become conventional in these articles to suppose that rivals using the platform sit on the opposite ends of the Hotelling line.

²See, for instance, Brito et al. (2013, 2014), Fudickar (2015), and Calzada and Tselekounis (2018).

Hotelling setup without a platform player. By comparison, if rival sellers have the first move, as long as consumers value the good well above (at least double) the Hotelling transportation cost, we show that in a symmetric equilibrium, sellers may set the same price that they would offer in a full coverage equilibrium without a platform.³ We conclude by discussing the advantages and reasonableness of these alternative move orders.

2 Model

Suppose that a platform connects consumers to two competing sellers, A and B . Seller A is located at point 0 and Seller B is located at point 1 of a unit line with a mass 1 of consumers uniformly distributed over the line on $[0, 1]$. Suppose that sellers are symmetrically differentiated with marginal cost zero. A consumer located at point $x \in [0, 1]$ on the line faces “transportation cost” tx when buying a product from Seller A and cost $t(1 - x)$ when buying from Seller B , where $t > 0$ indicates the degree of product differentiation. We consider games with two different move orders: in Section 2.1 we consider a game in which the platform first sets a market entrance fee followed by sellers’ pricing decisions; in Section 2.2 we consider a game in which the sellers move first followed by the platform.

2.1 Platform with First-Mover Advantage

Consider a game in which the platform first charges consumers a fee H for access to the market, after which Sellers A and B simultaneously set respective unit prices p_A and p_B . Suppose that consumers purchase a product from at most one seller. A consumer located at x obtains utility

$$U = \begin{cases} v - H - tx - p_A & \text{from Seller } A, \\ v - H - t(1 - x) - p_B & \text{from Seller } B. \end{cases} \quad (1)$$

Assuming full market coverage for a given H , the marginal consumer x^* who is indifferent between Sellers A and B satisfies:

$$v - tx^* - p_A - H = v - t(1 - x^*) - p_B - H. \quad (2)$$

³Even when this does not lead to a qualitative change, the reversal can have economically meaningful welfare implications. For example, reversing the order of moves used by Gautier and Somogyi (2020)—who, in an extension of their work identify an equilibrium outcome where asymmetric sellers move before the platform—leaves consumers better off than if sellers moved first.

Solving for x^* yields:

$$x^* = \frac{p_B - p_A + t}{2t}. \quad (3)$$

A seemingly intuitive approach to solve for equilibrium is to work backward, with Sellers A and B first setting prices to maximize profits, respectively $p_A x^*$ and $p_B(1 - x^*)$, to yield prices $p_A^* = p_B^* = t$ and equally splitting the market (i.e., $x^* = 1/2$).⁴ Substituting p_A^* , p_B^* , and $x^* = 1/2$ into consumers' utility of choosing Seller i , $U_i(x, p_i, H)$, yields the indifference utility:

$$U_i(x^*, p_i^*, H) = v - 3t/2 - H. \quad (4)$$

Assuming the platform cannot discriminate across consumers when charging its market entrance fee, it sets the fee to $H^* = v - 3t/2$ and extracts the indifferent consumer's entire surplus. The platform's profit equals $\pi_p^* = v - 3t/2$ and the sellers' profits are $\pi_A^* = \pi_B^* = t/2$.

Unfortunately, the prior, seemingly standard, Hotelling methodology incorrectly ignores the platform's first-mover advantage, leading to seller prices above the level that prevails in equilibrium. The omission results from a failure to consider that sellers face a constrained optimization problem and that in the correct solution, seller prices are bounded by the constraint that the indifferent consumer's utility equals zero conditional on the equilibrium value of H , rather than solving the problem for arbitrary H assuming that there is full market coverage. Thus, the platform with the first move extracts consumer surplus that otherwise would be captured by sellers by setting the optimal H , which puts downward pressure on sellers' prices.

To obtain the correct solution, we instead solve for the full market coverage equilibrium by taking the limit of the solution under partial coverage as it approaches full coverage. Consider a set of parameters that leads to partial market coverage and define x_i as the consumer who is indifferent between Seller $i \in \{A, B\}$ and leaving the market. Demand for Sellers A and B can respectively be specified as:

$$x_A = \frac{v - p_A - H}{t}, \quad 1 - x_B = \frac{v - p_B - H}{t}, \quad (5)$$

where $x_A \leq x_B$. Under partial coverage, for any H , Seller i solves:

$$\max_{p_i} p_i \left(\frac{v - p_i - H}{t} \right) \text{ s.t. } x_A \leq x_B. \quad (6)$$

When the constraint in Expression (6) does not bind, simultaneous solutions to sellers' first-order

⁴This is the standard Hotelling solution with linear transportation cost and no marginal production cost. It is readily verified that the matrix of second order price partial derivatives of profits is negative definite, so that the equilibrium is locally strictly stable.

conditions yield equilibrium prices $\hat{p}_A(H) = \hat{p}_B(H) = (v - H)/2$ and equilibrium quantities $\hat{x}_A(H) = 1 - \hat{x}_B(H) = (v - H)/2t$. Given $\hat{x}_A(H)$ and $\hat{x}_B(H)$, the platform sets H to solve:

$$\max_H H \left(\frac{v - H}{t} \right) \text{ s.t. } U_i(\cdot) \geq 0. \quad (7)$$

At the sellers' equilibrium prices and quantities, $U_i(\cdot) = 0$ for all values of H , so that the constraint can be ignored. Solving the first-order condition for the equilibrium market entrance fee yields $\hat{H} = v/2$, $\hat{p}_A = \hat{p}_B = v/4$, and $\hat{x}_A = 1 - \hat{x}_B = v/4t$.

If we assume that $\hat{x}_A(H) = \hat{x}_B(H) = x^* = 1/2$, which is the limiting case for the equilibrium quantities that solve Expression (6) as coverage increases from partial to full, then it must be that $H = v - t$ and $\hat{p}_A = \hat{p}_B = t/2$.

Our assumption that $\hat{x}_A \leq \hat{x}_B$ implies that $v \leq 2t$. A common approach employed when using Hotelling style models is to then assume that v is sufficiently large to entail full market coverage and to rely on Equations (2) and (3) to solve for equilibrium. As we next show, this is where seller prices are constrained by the indifferent consumer's utility. Let us then assume that $v > 2t$ and consequently $x_A = x_B$. Then, given H , Seller $i, i \neq j \in \{A, B\}$ solves:

$$\max_{p_i} p_i \left(\frac{p_j - p_i + t}{2t} \right) \text{ s.t. } U_i(\cdot) = v - tx^* - p_A - H \geq 0. \quad (8)$$

If the constraint in Expression (8) did not bind, then as we know from Equations (2) through (4), $x_A = x_B = x^* = 1/2$, $p_A^* = p_B^* = t$, and $H^* = v - 3t/2$. However, if it could continue to sell to the entire market, the platform would clearly be better off by setting $H = v - t$, which would violate the constraint in Expression (8). In this case, we already know that $\hat{p}_A = \hat{p}_B = t/2$ is a candidate pair of equilibrium prices and as we show in Proposition 1, sellers can do no better. Moreover, when $v > 2t$, the platform cannot profitably deviate to an H higher than $v - t$ (a lower H is suboptimal because it does not increase the number of customers).

Proposition 1. *Suppose that after the platform charges market entrance fee H , sellers set prices for a product and each consumer purchases a product from a single seller. Then, if $v > 2t$, in the unique subgame perfect Nash equilibrium outcome of this game, the platform sets equilibrium market entrance fee $\hat{H} = v - t$ and sellers fully cover and equally split the market with equilibrium prices $\hat{p}_A = \hat{p}_B = t/2$.*

Proof. The proof of Proposition 1 proceeds in three parts. We first show that given $H = v - t$, and rival price $t/2$, a seller cannot profitably deviate from a price of $t/2$. Next, we show that given sellers' best response functions, the platform cannot do any better than $H = v - t$. To complete the proof,

we show that the symmetric seller pricing outcome is unique.

Suppose that $H = v - t$. If we substitute $p_j = t/2$ into the objective function in Expression (8) and solve the maximization problem for p_i , it becomes evident that the constraint binds—the unconstrained optimum price of $p_i = 3t/4$ would leave the indifferent consumer with negative utility from consuming a product from Seller i —and because the objective function is concave in p_i , $t/2$ is the best that Seller i can do under full coverage (that is, there is no profitable deviation to a lower price). Thus, if Seller i raises price in response to $p_j = t/2$, it must be that $x_A < x_B$ (because at $\hat{H} = v - t$ and $\hat{p}_i = \hat{p}_j = t/2$, the indifferent consumer gets zero utility). However, by substituting $\hat{H} = v - t$ into Expression (6), we see that the price that maximizes each Seller’s profit equals $t/2$ —the price that prevails under full coverage—so profits cannot be increased via partial coverage.

Next, suppose that sellers price according to their best responses to \hat{H} . We show that in this case, the platform cannot do any better than $H = v - t$. First, from the negative definiteness of the matrix of second order price partial derivatives of seller profits given in Expression (8), we know that when $v > 2t$, any market entrance fee below $H = v - t$ leads to full coverage. But this means that any market entrance fee below $H = v - t$ cannot be profit maximizing.

Suppose to the contrary that $H = v - t + \varepsilon$ for some $\varepsilon > 0$. From Expression (8) we know that were the sellers to then set prices to achieve full market coverage, they would be bound by the non-negative utility constraint and set prices to $\bar{p}_A(x) = t(1 - x) - \varepsilon$ and $\bar{p}_B(x) = tx - \varepsilon$. We proceed as follows: we first show that when $H = v - t + \varepsilon$, there is no set of equilibrium seller prices that would lead to full coverage; we then show that given the partial coverage equilibrium that prevails, the platform is better off lowering its fee to $H = v - t$.

Suppose that under full coverage, $x \geq 1/2$. In this case, if Seller B sets price $\bar{p}_B(x) = tx - \varepsilon$ under full coverage, substituting $\bar{p}_B(x)$ into the objective function in Expression (8) and solving Seller A ’s maximization problem for p_A , it becomes evident that the constraint binds—the unconstrained optimum price of $p_A = [t(1 + x) - \varepsilon]/2$ is strictly higher than $\bar{p}_A(x)$ for $x \geq 1/2$ and would lead the indifferent consumer with negative utility from consuming a product from Seller A —and because the objective function is concave in p_A , $\bar{p}_A(x)$ is the best that Seller A can do under full coverage. Seller A ’s profit when sellers set prices $\bar{p}_A(x)$ and $\bar{p}_B(x)$ equals $\bar{\pi}_A(x) = [t(1 - x) - \varepsilon]x$, which is highest when $x = 1/2$ (note that $\bar{\pi}_A(1/2) = (t/2 - \varepsilon)/2 > \bar{\pi}_A(x)$ for any $x \in (1/2, 1]$). Under partial coverage, from Expression (6), we can see that $\tilde{p}_A = (t - \varepsilon)/2$ and profits for Seller A are:⁵

⁵We note that starting from a full coverage equilibrium with $x \geq 1/2$, when Seller A increases its price above $\bar{p}_A(x)$, x does not shift left toward zero because at $\bar{p}_B(x)$, the indifferent consumer receives zero utility from consuming a product from Seller B and instead becomes indifferent between consuming from Seller B and exiting the market.

$$\tilde{\pi}_A = \frac{t - \varepsilon}{2} \left[\frac{v - (t - \varepsilon)/2 - v + t - \varepsilon}{t} \right] = \frac{t}{4} - \frac{\varepsilon}{2} + \frac{\varepsilon^2}{4t} > \frac{t}{4} - \frac{\varepsilon}{2} = \bar{\pi}_A(1/2). \quad (9)$$

Thus, for any $x \in [1/2, 1]$, Seller A deviates to partial coverage. Similarly, it can be shown that for any $x \in [0, 1/2]$, Seller B would deviate to partial coverage such that $\tilde{\pi}_B = \tilde{\pi}_A$. Moreover, by checking second order conditions, it is readily verified that the symmetric partial coverage equilibrium that prevails when $H = v - t + \varepsilon$ is unique.

Expression (9) implies that when the platform sets $H = v - t + \varepsilon$, sellers set high enough prices that $x_A < x_B$ regardless of v . Substituting $H = v - t + \varepsilon$ into the objective function in Expression (7), we see that under partial coverage, the platform earns:

$$\tilde{\pi}_{\text{platform}} = (v - t + \varepsilon) \left(\frac{t - \varepsilon}{t} \right). \quad (10)$$

Comparing $\tilde{\pi}_{\text{platform}}$ with $\pi_{\text{platform}}^* = \hat{H}$ yields:

$$\pi_{\text{platform}}^* = v - t > (v - t + \varepsilon) \left(\frac{t - \varepsilon}{t} \right) = \tilde{\pi}_{\text{platform}} \iff 0 > (2t - v)\varepsilon - \varepsilon^2. \quad (11)$$

Because $0 > 2t - v$ by assumption, the inequality on the right always holds, so that it is unprofitable for the platform to set its market entrance fee above $v - t$. By setting $\varepsilon = 0$ in Equation (11), we can see that when $H = v - t$ in equilibrium, the equilibrium product prices are given by $\hat{p}_A(x) = t(1 - x)$ and $\hat{p}_B(x) = tx$ under full coverage.⁶ At these prices, Seller A earns profit $\hat{\pi}_A(x) = t(1 - x)x$, which is maximized at $x = 1/2$ and leads to $\hat{\pi}_A(1/2) = t/4$.

However, in this case, for any $x > 1/2$, Seller A wishes to deviate to a higher price (and a partial coverage equilibrium). Under partial coverage, from Expression (6), we know that Seller A can earn $\tilde{\pi}_A = t/4$ by deviating to $\tilde{p}_A = t/2$, which is greater than $\hat{\pi}_A(x) = t(1 - x)x$ when $x > 1/2$. Similarly, we can show that Seller B would wish to deviate to a higher price for any $x < 1/2$. Thus, the equilibrium set of prices given in Proposition 1 is the unique full coverage subgame perfect Nash outcome. \square

Figure 1 graphs inverse demand for Seller A when the platform and Seller B price according to the equilibrium in Proposition 1. The figure depicts full and partial coverage inverse demand. From Equation (3), full coverage inverse demand for Seller A is given by $p_A = p_B + t - 2tx_A$ and from

⁶Suppose that $x > 1/2$. Substituting $\hat{p}_B(x) = tx$ into the objective function in Expression (8) and solving Seller A 's maximization problem for p_A , it becomes evident that the constraint binds—the unconstrained optimum price of $p_A(x) = t(x + 1)/2$ is strictly higher than $\hat{p}_A(x) = t(1 - x)$ for $x \geq 1/2$ and would lead the indifferent consumer with negative utility from consuming a product from Seller A —and because the objective function is concave in p_A , $\hat{p}_A(x)$ is the best that Seller A can do under full coverage.

Equation (5), partial coverage inverse demand for Seller A is given by $p_A = v - H - tx_A$. When substituting the equilibrium values— $\hat{p}_B = t/2$ and $\hat{H} = v - t$ —into the inverse demand curves, it can be seen that the vertical intercept under full coverage is $\hat{p}_B + t = 3t/2$ and the vertical intercept under partial coverage is $v - \hat{H} = t$. Additionally, as the figure shows, inverse demand has an outward kink, above which the slope equals $-t$, and below which the slope equals $-2t$. Given \hat{H} , above the kink, prices are sufficiently high to lead to partial coverage, whereas below it, full coverage emerges in equilibrium.

By setting H , the platform determines the location of the kink and the seller price above (below) which partial (full) coverage would result. If, alternatively, H were set to $H = v - 3t/2$ or lower, partial coverage would not occur. However, as long as a value of H above $v - 3t/2$ still results in full coverage, the platform would prefer to raise its market entrance fee, causing a kink to form. At the platform's optimal market entrance fee, the kink occurs at $\hat{p}_A = t/2$. Thus, Seller A faces elasticity higher than -1 above the kink and elasticity lower than $-1/2$ below it, maximizing total revenue (which, here, equals total profit) by pricing at the kink.

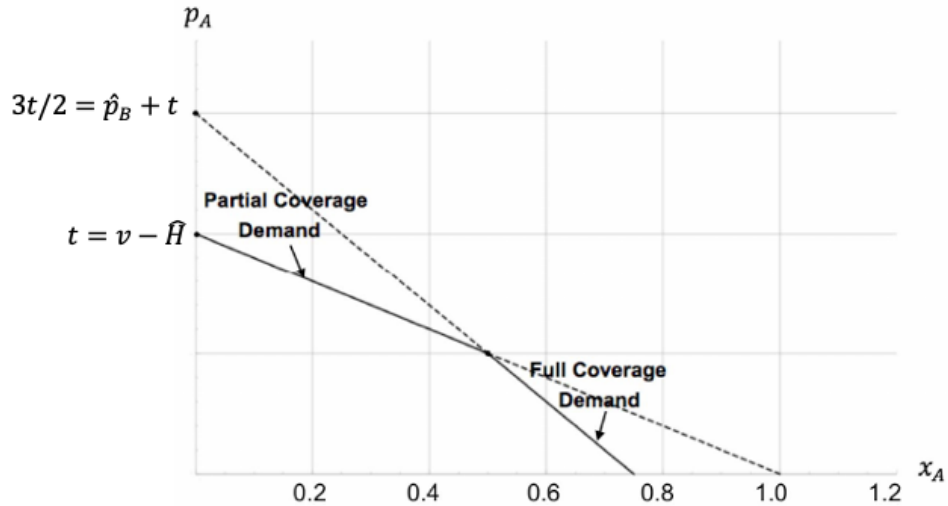


Figure 1: Seller A Inverse Demand at \hat{H} and \hat{p}_B

Because the kink in Figure 1 leads to full coverage, the platform clearly would not wish to set H lower than \hat{H} . Suppose instead that the platform were to raise H slightly, to $\bar{H} > \hat{H}$, shifting down the partial coverage inverse demand curve. Then, Seller A would view the region immediately above the ensuing kink as inelastic, best responding with a price that leads to partial coverage. In contrast, because the platform loses twice as many customers as the sellers do at prices above the kink, it views its own demand as elastic at \bar{H} and prefers to keep its fee at \hat{H} .

2.2 Sellers with First-Mover Advantage

Proposition 1 states that the full coverage outcome derived using Equations (2) through (4) incorrectly leads to a platform market entrance fee that is too low and seller prices that are too high. We next consider how altering the timing of the game can restore the outcome obtained in these equations. That is, suppose instead that the sellers set prices ahead of the platform market entrance fee. Clearly in this case, assuming full coverage, the platform cannot claim a first-mover advantage, and consequently, Equations (2) to (4) apply. Thus, it only remains to check whether or not the platform has an incentive to deviate to an H that leads to partial coverage.

In order to home in on the parameter space under which full coverage is preferred, let us again start by solving for the partial coverage equilibrium. Thus, consider the partial coverage demands given in Expression (5) and suppose that $x_A < x_B$. Working backward, for any p_A and p_B , the platform solves:

$$\max_H H(x_A + 1 - x_B) = H \left(\frac{2v - p_A - p_B - 2H}{t} \right) \quad \text{s.t.} \quad x_A < x_B. \quad (12)$$

The platform's first order condition yields equilibrium market entrance fee $H^* = (2v - p_A - p_B)/4$ and equilibrium quantities $x_A^* = (2v - 3p_A + p_B)/(4t)$ and $1 - x_B^* = (2v - 3p_B + p_A)/(4t)$.⁷ Given x_A^* and x_B^* , sellers simultaneously set p_A and p_B to maximize profit. Seller $i, i \neq j \in \{A, B\}$ solves:

$$\max_{p_i} p_i \left(\frac{2v - 3p_i + p_j}{4t} \right) \quad \text{s.t.} \quad U_i(\cdot) \geq 0. \quad (13)$$

Simultaneously solving sellers' first order conditions yields equilibrium prices $p_A^* = p_B^* = 2v/5$.⁸ Accordingly, the market entrance fee equals $H^* = 3v/10$ and $x_A^* = 1 - x_B^* = 3v/(10t)$. Thus, partial coverage ($x_A^* < x_B^*$) occurs if and only if $v < 5t/3$.

Because we are interested in full market coverage, let us assume that $v \geq 5t/3$. As Proposition 2 states, v needs to be somewhat higher than $5t/3$ in order for $p_A^* = p_B^* = t$, the equilibrium prices stemming from Equation (3), to come about. Indeed, as it turns out, the same level of $v, v \geq 2t$, that results in full coverage in a game where the platform moves first, permits the canonical Hotelling outcome computed in Equations (2) to (4) to arise in equilibrium when the sellers move first instead.

Proposition 2. *Suppose that after sellers simultaneously set prices p_A and p_B , the platform sets market entrance fee H and each consumer purchases a product from a single seller. Then, if $v \in [5t/3, 2t]$, in the unique symmetric equilibrium, the optimal market entrance fee equals $H^* = t/2$ and*

⁷It is easily verified that the second order condition is satisfied.

⁸Again, the matrix of second order price partial derivatives of profits is negative definite.

sellers charge prices $p_A^* = p_B^* = v - t$ and equally split the market. If $v \geq 2t$, then market entrance fee $H^* = v - 3t/2$ and seller prices $p_A^* = p_B^* = t$ comprise a full coverage equilibrium outcome.

Proof. The proof follows a similar approach to that of Proposition 1, but in two parts, one for $v \in [5t/3, 2t]$ and the other for $v > 2t$. For $v \in [5t/3, 2t]$, we first show that the highest market entrance fee that maintains full coverage is an optimal response to sellers' equilibrium prices and that given the platform's optimal market entrance fee, neither seller has an incentive to deviate to a price that leads to partial coverage. From the computation of partial equilibrium prices (Expressions (12) and (13)), we know that when $v = 5t/3$, $p_A^* = p_B^* = 2v/5 = v - t$.⁹ As the seller prices that prevail in the limit of the equilibrium under partial coverage as it approaches full coverage, $p_A^* = p_B^* = v - t$ represent a candidate pair of full coverage equilibrium prices. Under full coverage, given seller prices, the platform maximizes H subject to $U_A(\cdot) = U_B(\cdot) = 0$, which in the case that $p_A^* = p_B^* = v - t$ leads to $H^* = (2v - p_A - p_B)/4 = t/2$. At $H^* = t/2$, the platform has no desire to lower its price, whereas it brings about partial coverage by raising it. From Expression (5) it follows that the platform's partial coverage demand when $p_A^* = p_B^* = v - t$ equals $2(t - H)/t$. Thus, the platform maximizes its partial coverage profit by choosing $H = t/2$, which brings it back to the full coverage outcome.

We next want to know whether either seller has an incentive to deviate given its rival's price and the optimal platform market entrance fee. Suppose that $p_A = v - t + \varepsilon$ for $\varepsilon \neq 0$ given $p_B = v - t$. Note that Seller A raises its price if $\varepsilon > 0$ and lowers it if $\varepsilon < 0$. Suppose first that $p_A = v - t + \varepsilon$ and $p_B = v - t$ leads to full coverage, so that the platform maximizes H subject to $U_A(\cdot) = U_B(\cdot) = 0$. Then, $\bar{\pi}_{platform} = \bar{H} = (t - \varepsilon)/2$. By raising its price above $(t - \varepsilon)/2$, the platform brings about partial coverage. Under partial coverage, the platform solves:

$$\max_H H \left(\frac{2t - 2H - \varepsilon}{t} \right) \quad \text{s.t.} \quad x_A < x_B, \quad (14)$$

which leads to a market entrance fee of $\tilde{H} = t/2 - \varepsilon/4$. Observe that $\tilde{H} > \bar{H}$ only when $\varepsilon > 0$, implying that partial coverage does not emerge when $\varepsilon < 0$. We next consider the cases $\varepsilon > 0$ and $\varepsilon < 0$ in turn.

Suppose that $\varepsilon > 0$. The platform's profit from deviating to $\tilde{H} = t/2 - \varepsilon/4$ equals $\tilde{\pi}_{platform} = (2t - \varepsilon)^2/(8t)$. Thus, $\tilde{\pi}_{platform} > \bar{\pi}_{platform}$ holds if and only if $\varepsilon^2/(8t) > 0$ which is always the case. Thus, the platform optimizes by deviating to partial coverage. Substituting $p_A = v - t + \varepsilon$ and $p_B = v - t$ into partial coverage profit Expression (6) yields $\tilde{\pi}_A = (2t - 3\varepsilon)(v - t + \varepsilon)/(4t)$. Seller A deviates from $p_A^* = v - t$ to $p_A = v - t + \varepsilon$ only if $\tilde{\pi}_A > \pi_A^* = (v - t)/2$. However, because $\pi_A^* > \tilde{\pi}_A$

⁹When $x_A^* = x_B^* = x^* = 1/2$ and $p_A^* = p_B^* = p^*$, $x^* = (2v - 3p^* + p^*)/(4t) = 1/2$ implies that $p^* = v - t$.

if and only if $(3v - 5t)\varepsilon + 3\varepsilon^2 > 0$ and by assumption, $3v - 5t > 0$, Seller A does not wish to price higher than $p_A^* = v - t$. By the same logic, Seller B does not have a profitable deviation to a price higher than $v - t$.

Suppose now that $\varepsilon < 0$. The lower Seller A price maintains full coverage and from $\tilde{H} < \bar{H}$ (for $\varepsilon < 0$) above, we know that the platform would not wish to deviate to a higher price. Thus, when $\varepsilon < 0$, the platform optimally sets its price to $\bar{H} = (t - \varepsilon)/2$ and achieves full coverage. To see whether Seller A has an incentive to deviate to begin with, we compare $\pi_A^* = (v - t)/2$ to $\bar{\pi}_A$, Seller A 's full coverage deviation profit. It can be verified that $\pi_A^* \geq \bar{\pi}_A$ holds if and only if $\varepsilon \leq 2t - v$. Because $v \in [5t/3, 2t]$ and $\varepsilon < 0$, π_A^* is always greater than or equal to $\bar{\pi}_A$, so that neither seller has a profitable deviation. Therefore, we have shown that $p_A^* = p_B^* = v - t$ and $H^* = t/2$ represent a full coverage equilibrium outcome for $v \in [5t/3, 2t]$. However, when $v \geq 2t$, there is always an ε close enough to zero such that each seller wishes to deviate unless $v = 2t$. Note that because $v - t = t$ when $v = 2t$, $p_A^* = p_B^* = t$ is our symmetric equilibrium candidate for $v > 2t$.

Next, we show that there is a profitable deviation to partial coverage by either the platform or one of the sellers from any other symmetric set of prices. Suppose instead that $p_A^* = p_B^*$ at some price other than $v - t$. Suppose first that $p_A^* = p_B^* = v - t + \varepsilon/2$ for some $\varepsilon > 0$. Then, the best that the platform can do under full coverage is $H = (t - \varepsilon)/2$. By deviating to a higher price, the platform causes partial coverage. In this case, the platform solves Expression (14), which leads to a market entrance fee of $H = t/2 - \varepsilon/4$. We know that this will be more profitable for the platform, so that $p_A^* = p_B^* = v - t + \varepsilon/2$ cannot be a full coverage outcome when $\varepsilon > 0$.

There is another case in which $p_A^* = p_B^* = v - t + \varepsilon$ for some $\varepsilon < 0$ leads to full coverage. Suppose that Seller B deviates to $\bar{p}_B = v - t$. We know that in this case, under full coverage, the platform sets $\bar{H} = (t - \varepsilon)/2$ and does not wish to deviate to partial coverage. Comparing seller profit in the proposed equilibrium, $\pi_B^* = (v - t + \varepsilon)/2$, to its deviation profit, $\bar{\pi}_B = (v - t)(\varepsilon + t)/2t$, we have $\bar{\pi}_B \geq \pi_B^*$ if and only if $2t \geq v$, which holds by assumption. Thus $p_A^* = p_B^* = v - t$ is the only symmetric full coverage outcome for $v \in [5t/3, 2t]$.

For $v > 2t$, it remains to show that $p_A^* = p_B^* = t$ and $H^* = v - 3t/2$ represent a full coverage equilibrium outcome. We first consider the platform's deviation incentive. A lower H is clearly suboptimal whereas a higher H leads to partial coverage. In the latter case, when $p_A^* = p_B^* = t$, the platform maximizes $H(2v - 2t - 2H)/t$ with respect to H , which yields $\tilde{H} = (v - t)/2$. However, $\tilde{H} < H^*$ whenever $2t < v$, meaning that the platform does not have a profitable deviation to partial coverage. Moreover, from Equations (2) and (3) in the game where the platform moves first, we know that $p_A = p_B = t$ represents a set of mutual seller best responses, so that $p_A^* = p_B^* = t$ and

$H^* = v - 3t/2$ indeed represent a full coverage equilibrium in the game where the sellers move first whenever $v \geq 2t$. □

When v surpasses a certain threshold (in this case $5t/3$), a full coverage equilibrium exists. However, when sellers price at t in equilibrium, if v is not high enough, the platform does not get sufficiently compensated under full coverage and wishes to deviate to partial coverage by setting a higher market entrance fee. Once v reaches $2t$, by setting their prices to t , the sellers nevertheless leave enough consumer surplus to the platform to keep it from wanting to deviate to a higher price.

3 Conclusion

It is common to assume that platforms move ahead of sellers that rely on a platform when studying platform markets. This has been the case when sellers compete on a Hotelling line (Brito et al., 2013, 2014; Baranes, 2014; Fudickar, 2015; Calzada and Tselekounis, 2018) as well as in alternative models of seller competition (Economides and Hermalin, 2015; Jeitschko et al., 2020).¹⁰ Our work highlights how this move order not only changes the equilibrium outcome compared to a Hotelling game without a platform,¹¹ but also requires a non-standard solution approach. By comparison, when sellers move first and consumers value the good sufficiently above their Hotelling transportation cost, the solution and solution concept resemble those if there were no platform player.

In this sense, our work offers a road map for the appropriate application of a workhorse model in industrial organization. Specifically, the model in the paper suggests criteria that show how to model the platform market with full coverage in a correct way. If researchers do not have a specific reason for preferring a platform to move first, then assuming that sellers move first offers the advantage of tractability, especially if there are other features of the game that would complicate solving for equilibrium under the alternative move order. In many instances, the product market may well have developed before a platform of interest (such as an Internet service provider or club warehouse) came to exist, so if the associated prices are assumed to be stable or are fixed, then it may be sensible to suppose that sellers have the first move in a game in which their products or services are offered through a platform that later enters the market.

¹⁰Some of these models also break up consumers' decisions by requiring consumers to decide whether or not to participate in the market after the platform sets its market entrance fee, but before sellers set their prices. This turns out to be important when consumers are homogenous—requiring homogenous consumers to make their participation decision before sellers set prices admits a subgame perfect Nash equilibrium strategy in which seller prices are invariant to market entrance fees off the equilibrium path—but does not substantively alter the game in the Hotelling framework.

¹¹It is well established that players act strategically taking move order into account, with, for instance, first-movers pricing aggressively to deter entry (Etro, 2008), or, in the platform literature, second-movers relying on price-discrimination to overcome a move disadvantage (Jullien, 2011).

Conversely, there may be a compelling reason to believe that a platform has the first move. For example, paying an Internet platform may be its own source of utility because the platform offers broad access to the Internet. If such access is more valuable than the incremental value from additional paid content, then it might make more sense to view competing paid content providers as having the second move.

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