Quality Differentiation and Optimal Pricing Strategy in Multi-sided Markets*

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Abstract

This paper analyzes the generalized quality differentiation model in multi-sided markets with positive externalities, which leads to new insights into the optimal pricing structure of the firm. We find that quality differentiation for buyers affects not only the side involving differentiation but also the other side due to cross-side network externalities, thereby affecting the pricing structure of multi-sided firms. In addition, quality differentiation affects the strategic relationships among all the choice variables for the platform, enabling the platform to strategically use quality differentiation to increase its profits.

Keywords: Multi-sided Market; Quality Differentiation; Platform Business Strategies
JEL Classification Numbers: D43; L11; L42

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1 Introduction

There has been an increasing shift toward multi-sided firms in many industries. Early work on multi-sided platforms was introduced by Armstrong (2006), Caillaud and Jullien (2003), Parker and Van Alstyne (2005), and Rochet and Tirole (2003). As many previous studies emphasize, it is important to establish how multi-sided platforms differ from typical one-sided firms and how such differences lead to new business implications. Many markets that have traditionally featured one-sided firms now feature more two-sided firms due to advanced technology; for example, the taxi industry exclusively involved one-sided firms before Uber appeared. Given this shift toward multi-sided business, the main strategic choice that we analyze in this paper is the choice of the quality of interaction between buyers on two different sides. We find that in addition to the price elasticity on the two sides, the quality of interaction also influences the price ratio between the two sides. Additionally, if the platform uses quality information to charge different prices on one side, then the optimal price on the other side changes as well.

We focus on platforms that enable interaction between two sides, represented as buyers and developers. The term "interaction" covers various traditional interactions, including exchanges between application developers and application buyers on software platforms such as computers (e.g., Apple, Microsoft), mobile devices (e.g., iPhone, Samsung), and video games (e.g., Sony PlayStation, Xbox). Interaction also refers to the interactions observed in auction houses and on Internet sites in person-to-business transactions (e.g., Amazon). These markets usually have more than one type of quality access on the buyer's side. For example, in the video game industry, Sony launched PlayStation *Pro* (high-quality) together with *Slim* (low-quality). Amazon offers buyers two types of quality access: basic (low-quality) access is free, while premium access (*Prime* membership, which is high-quality) is the paid service. It is easy to see that the interactions or exchanges through the high-quality access have better quality—for example, PlayStation *Pro* supports 4K resolution, and *Prime* provides two-day shipping. Many ride-sharing services offer differentiated quality tiers for customers. For example, Uber provides riders with multiple types of quality services. Although each region may have different availability, there is basically a low-quality service at a less expensive price (e.g., *Uber X, Uber Pool*) and a high-quality service at a slightly more expensive price (e.g., *Uber XL, Uber Select, Uber Black*).

From these examples of quality differentiation by firms serving two sides, we can see that the platform's different quality provisions on the buyer's side affect the per buyer profit for developers. For example, if Sony upgrades PlayStation in terms of display quality, those game console buyers become more likely to subscribe to video game providers, which implies that there are positive effects of Sony's quality improvement on game developers' revenues. In this regard, our paper provides the business implications of quality differentiation in the multi-sided platform market by building a simple model of two-sided monopolists. In the model, the quality access for one side also affects the utility derived by the other side, thereby extending Rochet and Tirole (2003) by accommodating the effect of quality on cross-side network externality.

We first emphasize that by providing more quality choices, even if the quality differentiation is small, the platform can earn more profits. Given that buyers are heterogeneous in their valuation of product quality, the platform is incentivized to offer different levels of product quality at different prices to extract more rents from buyers. More importantly, we endogenize the impact of the quality of the platform's product or service used by the buyer's side on the profit made by developers. We find that optimal pricing depends on the quality of the platform's product or service. In particular, we show that the quality of the platform changes the pricing ratio on both sides due to the multi-sidedness of the platform. Based on this finding, we present managerial implications about the quality of the product to be provided such that access quality for buyers not only affects the price on the buyers's side but also influences the price on the developer's side.

Additionally, in the case of two qualities, our paper sheds light on how prices might change when quality differentiation affects profits for both buyers and developers. We start by examining the outcome in a specific scenario where quality differentiation has no direct impact on developer profits. Here, we observe that as quality differentiation increases profit for buyers, prices on the developer's side decrease.

However, in the broader and more nuanced scenario where quality differentiation affects both sides, we discover that it might be advantageous for the platform to set a higher price for developers when quality differentiation occurs on the buyer's side. Specifically, we identify the condition under which quality differentiation on the buyer's side leads to an increase in developer prices: if offering higher quality allows the platform to extract more rent from developers than from buyers, then it is optimal to increase the price for developers in the two-quality case compared to the one-quality case. This is due to significant network externalities, wherein the profit-enhancing effects are more pronounced on the side not experiencing differentiation.

Furthermore, we show that the price of the low-quality product on the buyer's side and the fee charged on the developer's side move in the opposite direction upon quality differentiation on the buyer's side. This finding suggests that the platform can optimally lower the basic quality product price for buyers, while raising the price for developers upon quality differentiation. Thus, the introduction of a high-quality product on one side alters the pricing dynamics between the two sides via network effects.

Based on the model predictions, we discuss some business implications for how quality differentiation on one side of the market reshapes the platform's profit-maximizing strategies via cross-side externalities.

Related literature There is a broad literature on the corresponding issue of a monopolistic firm seeking to maximize profits through quality-differentiated products in one-sided standard markets. The formative papers are those by Spence (1975), Mussa and Rosen (1978), and Maskin and Riley (1984). We generalize this problem by varying the number of sides served by the firm.

This paper is also related to the literature on pricing structure in markets with multi-sided firms (e.g., Rochet and Tirole, 2003; Parker and Van Alstyne, 2005; Armstrong, 2006b; Reisinger, 2010). A major difference in this paper is that we consider a three-way interaction among prices for buyers and developers and quality choice for the optimal pricing structure in multi-sided firms. In addition to being broadly related to the literature on pricing in multi-sided markets, papers on skewed pricing in multi-sided markets are closely related to our paper in terms of theoretical implications. Suarez and Cusumano (2008) discuss the platform's subsidy pricing strategy to attract greater buyer adoption, although they do not set up an economic model to confirm this strategy. Bolt and Tieman (2008), Schmalensee (2011), and Dou and Wu (2018) study skewed pricing strategies in two-sided markets, i.e., the subsidy and money sides. However, those papers do not consider forms of product differentiation, such as the quality differentiation examined in our paper, as a means of skewing prices. Additionally, Sridhar et al. (2011) focus on cross-market network effects in two-sided markets, as we do in this study; however, their focus is on empirical analysis of how optimal marketing investment allocation is affected by cross-market effects.

Regarding markets with multi-sided firms, few papers have focused on firms considering the quality of interaction on their platform. These papers study markets with negative network externalities and do not take into account how quality differentiation on one side affects the other side. Crampes and Haritchabalet (2009) examine the choice of offering a pay ads regime and no pay ads packages. Peitz and Valletti (2004) compare advertising intensity when media operators offer free services and when the subscription price is positive. Viecens (2006) is an exception because this study considers a setup with endogenous quality differentiation on two-sided platforms. However, the quality differentiation in the model takes a different form from ours in that she focuses on the quality provided by buyers on one side and not on the quality provided by the platform itself. Therefore, the results do not provide any implications for the platform's dynamic pricing structure.

Another quality-related aspect explored in the context of two-sided markets is the case in which buyers care about the quality of the other buyers with whom they interact, which is relevant for matching markets such as dating sites. Jeon et al. (2016) examine this problem in a platform setting.

Renato and Pavan (2016) consider this problem in a matching setup. Hagiu (2012) studies a model in which buyers value the average quality of other buyers. The setup in these papers, however, is different from that in ours in that we focus on the quality of interaction and not on the quality of buyers.

Overall, the previous literature on quality discrimination focuses on the quality of products sold on the platform, which affects the interaction between buyers, similar to our case. However, those studies do not consider quality differentiation as a cross-market business strategy that the platform can implement. Our paper aims to provide a basic model to endogenize the effect of quality. Given that the quality of the platform is an important feature that is observed in many multi-sided markets, our approach also provides an opportunity for extending the model to diverse types of platform markets (e.g., ride-sharing, e-commerce, and game platforms) involving quality differentiation.

Our analyses are also related to the literature on product differentiation in multi-sided markets in that quality differentiation is one form of product differentiation. Smet and Cayseele (2010) focus on product differentiation in platform markets, which still differs from our paper in that they do not account for its consequences for optimal pricing strategies. Additionally, our model also considers how quality access affects the buyer's interaction with the other side, namely, cross-side network externalities.

In summary, the literature has not focused on the choice of network quality differentiation in multi-sided markets with positive cross-network externalities. Therefore, our results on how multi-sided markets are combined with quality differentiation on a platform, given that multiple quality access on one side affects the revenue-maximizing prices of both or multiple sides, provide new insights into the related business, which is underexplored in the literature.

2 Model Setup

We model the interaction between buyers and developers. Economic value is created through the interaction between these two sides. We consider the case of a single firm providing a platform for interactions between buyers and developers. For instance, Sony PlayStation as a game console provides interactions between game buyers and developers. In addition to charging an access fee to use the game platform, the firm can also control the quality of the interaction by launching a new generation or releasing multiple quality tiers, e.g., PlayStation *Pro* vs. *Slim*.

Figure 1 displays the structure of the two-sided firm with quality differentiation on the buyer's side using PlayStation as an example. Buyers are able to access games only when they buy compatible devices, in this case, different generations of PlayStation; thus, purchasing a game console is equivalent to purchasing access to the game store. The two types of access quality offered on the buyer's side are PlayStation *Slim* (basic low-quality) and *Pro* (high-quality). For instance, if buyers purchase *Pro*, which provides 4K resolution, which does not come with *Slim*, *Pro* buyers obtain greater utility from playing the same games.

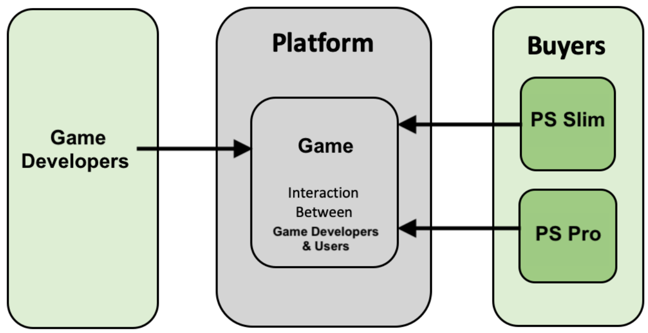

Figure 1: Two-sided firm with quality differentiation

Quality provision We consider two scenarios: with and without quality differentiation implemented by the platform. In the one-quality model, as described in Section 2.1, the platform does not differentiate the quality level offered to buyers; one quality level in this model is denoted as $q_l \in [0, 1]$. In the two-quality model, as in Section 2.2, the platform introduces a higher quality product in addition to the basic quality option, which is available in one-quality case. For instance, Sony operates the platform, i.e., PlayStation, to connect buyers and game developers: buyers join the platform by purchasing PlayStation (game console), whereas game developers sell games on the platform. If Sony offers PlayStation *Slim* only, i.e., the one-quality case, its buyers obtain utility from its quality measured by q_l. If Sony offers both (basic) PlayStation *Slim* and (upgraded) *Pro*, i.e., the two-quality case, the basic low quality of *Slim* is q_l, which is the same as before, and the high quality of *Pro* is measured by $q_h > q_l$.

Buyer's side Specifically, we model a two-sided monopolist firm that charges different prices on the buyer's side by offering two different types of access quality: $q_k \in \{q_l, q_h\}$, where k denotes the quality provision, either *l*ow or *h*igh. In this model, the buyer's gain from the platform is derived through interaction between users on two sides: buyers and developers. Both buyers and developers obtain utility from interacting with each other. The quality variable controls the gain from each such interaction. We assume that both buyers and developers are heterogeneous with respect to the per interaction or usage benefit. The usage or per interaction benefits are $b^u(q_k)$ for the buyer's side (for an individual buyer), where the superscript u denote the buyer side. For simplicity, we assume that

$b^u(q_k) = \mathcal{U}\alpha^u q_k$, where $\mathcal{U}$ represents the basic benefit for every buyer. This benefit is dependent on the quality of interaction; thus, the monopolist can control the benefit by choosing whether to offer higher quality access on the buyer's side. The term α^u denotes the heterogeneity among buyers; it follows a distribution function F^u with support on $[0,1]$ and density f^u. Additionally, each buyer pays a price for using the platform, which is denoted $P^u_{k,m}$, as a fixed fee, where $m \in \{1,2\}$ denotes the number of quality levels available on the buyer's side. For a buyer, the utility function is given as follows:

$$U^u_k = (\mathcal{U}\alpha^u q_k N^d) - P^u_{k,m}, \tag{1}$$

where $k \in \{l,h\}$ and $m \in \{1,2\}$. The buyer's utility is the benefit from each interaction with the other side, i.e., $\mathcal{U}\alpha^u q_k N^d$, subtracted by a fixed fee to access the platform $P^u_{k,m}$.[1] From the utility specification, it can be shown that the buyer with the highest benefit is that with $\alpha^u = 1$.

We assume that the access fee $P^u_{k,m}$ is a fixed fee. For example, Sony charges a fixed fee as the price for PlayStation *Slim* or *Pro*. However, as shown in Appendix A.3, the model with a usage-based per transaction fee, such as Uber, yields the same results as in the model with a fixed fee.[2] Thus, according to the equivalence, we use the per transaction price $p^u_{k,m} \equiv \frac{P^u_{k,m}}{N^d}$ instead of the fixed fee $P^u_{k,m}$ for the remainder of the analysis. The equation below shows the equivalence.[3]

$$U^u_k = (\mathcal{U}\alpha^u q_k N^d) - P^u_{k,m} \quad \Leftrightarrow \quad U^u_k = (\mathcal{U}\alpha^u q_k - p^u_{k,m})N^d. \tag{2}$$

The buyer's utility is the net benefit from each interaction with the other side, i.e., $\mathcal{U}\alpha^u q_k - p^u_{k,m}$, multiplied by the number of interactions, which is denoted N^d. Note that we omit the subscript k when referring to a scenario where only one quality is offered.

Developer's side We assume that the developer's side is also affected by the quality if access is given to the buyers. Thus, the usage or per interaction benefit is denoted as $b^d(q_k)$ for the developer's side, where the superscript d denotes the developer's side. We assume that $b^d = \mathcal{D}g(q_k)\alpha^d$, where $g(q_k)$ denotes the relative benefit extracted from buyers with q_k quality access. Additionally, α^d represents heterogeneity among developers with respect to the per usage benefit, which is also distributed by a distribution function F^d with support on [0,1] and density f^d.

Fee structure: We assume that the fee charged to the developer is multiplicatively separable in

[1] We assume here that every buyer interacts with every developer and that every developer interacts with every buyer. We can easily extend this to a model where the number of interactions is a function of the total number of developers, such as $\Gamma(N^d)$. The results are robust to such extensions.

[2] Note that the equivalence holds as buyers only extract benefit from the platform through interaction with the other side, which is the case in Uber or game consoles or Amazon.

[3] Note that the transformation in the equation assumes no coordination issue between buyers and developers, and it does not work when there are competing platforms.

quality factors and the overall fee charged on the developer side, implying that it can be written as the product of two fee parameters. First, there is a *price index*, denoted by p_m^d, where $m \in \{1, 2\}$ denotes the number of quality levels available on the buyer's side. Second, we introduce a fee scaling parameter, which we assume is equal to $g(q)$. Thus, the overall fee charged on the developer side is $g(q_k) \times p_m^d$. It is crucial to remember that in this model, the endogenous pricing mechanism available to the platform on the developer side is the price index p_m^d, hereby referred as the fee on developer side.

For a developer, the utility function is given as follows:

$$U^d = \begin{cases} [g(q_l) \times (\mathcal{D}\alpha^d - p_1^d)]N^u, & \text{if one-quality case;} \\ [g(q_l)N_l^u + g(q_h)N_h^u] \times [(\mathcal{D}\alpha^d - p_2^d)], & \text{if two-quality case,} \end{cases} \tag{3}$$

where $g(q_k) \in [0, 1]$ is increasing and concave in quality level. N_k^u denotes the number of buyers who demand products of quality k. To distinguish the number of buyers in one-quality case from two-quality case, we denote that in one-quality case as N^u without k subscript. This utility specification accommodates the effect of the platform's quality provision on the per buyer profit for developers. For example, if Sony upgrades PlayStation quality in terms of display quality, i.e., q_k increases, those game console buyers are more likely to make high in-game purchases or spend more on the game.[4] In this sense, $g(q_k)$ captures the positive effects of Sony's quality improvement on game developers' revenues. Additionally, $g(q_k)$ captures how much the platform can extract additional rents from game developers in the form of the platform access fee p_m^d by offering a higher quality product to the buyer's side.[5] The platform access fee charged to developers p_m^d can be interpreted in different ways. For example, Microsoft, as a game publisher/platform, that builds games for its own consoles, *Xbox*, often makes a contract with (second- or third-party) game developers. Upon a contract agreement, the publisher pays royalties to developers from the sale of games. At the same time, since the publisher covers costs for development, it takes a certain share of the royalties, approximately 10-20%. The size of the share that goes to the publisher can be regarded as the fee charged to developers in this case.

The developer's utility is the net benefit from each interaction with the other side multiplied by the number of interactions.[6]

[4]Note that the model setting allows a general interpretation of why revenue might increase for developers if buyers access the higher-quality platform; for example, it accommodates cases where better quality access means that the developer can charge buyers a higher price for the game, that buyers buy more in-game items or that buyers become more likely to subscribe to video game providers. All of these cases can be accommodated in our given model.

[5]Note that we assume that the quality level as well as the platform's ability to extract differentiated rent, captured by $g(q_k)$, are exogenous. We can extend the model to look at the platform maximizing these two additional parameters as well.

[6]In this model, we assume that the developer's side is affected only by the total number of buyers, not the type of

Remark 1. ***Pricing structure on the developer side*** *The fee p^d_m captures the endogenous pricing mechanism available to the platform on the developer side. As we will see in Section 3, the platform optimally decides the fee. Recall that we assume the fee scaling parameter $g(q)$ is exogenously determined. Note that in the one-quality case, the $g(q) \in [0, 1]$ is non-consequential, as p^d_1 can be re-scaled to reach optimal pricing for developers. Additionally, in the two-quality case, the $g(q)$ factor influences the differential pricing allowed on the developer side for high/low access buyers. We assume that $g(q)$ factor is exogenous, thereby restricting attention on the overall fee charged on the developer side. Further relaxing this assumption will lead to a different market setting, which is not addressed in this model.*

The platform The cost of the two-sided monopoly firm depends on the quality provided. The total cost of a transaction is given by $c(q_l) \geq 0$ for a transaction between a low-quality buyer and a developer and $c(q_h) \geq 0$ for a transaction between a high-quality buyer and a developer. In the PlayStation example, such cost differentiation captures the fact that better resolution for PlayStation *Pro* buyers is costlier than standard resolution for *Slim* buyers. We normalize the cost for developers $c^d = 0$. We analyze the nontrivial case in which $q_h > q_l$.

The demand on the buyer's and developer's sides is represented by D^u and D^d, respectively. In equilibrium, demand will be equal to the number of participants on each side, which means $D^u_k = N^u_k$, where $k \in \{l, h\}$, thereby $D^u = N^u$, and $D^d = N^d$. Given the equilibrium demands, we turn to the monopolistic platform's problem. The monopoly platform's problem can be written as follows:

- If one type of quality is offered:

$$\max_{p^d_1, p^u_1} \Pi = [p^u_1 - c(q_l) + g(q_l)p^d_1]D^u D^d. \tag{4}$$

- If two types of quality are offered:

$$\max_{p^d_2, p^u_{l,2}, p^u_{h,2}} \Pi = [p^u_{l,2} - c(q_l) + g(q_l)p^d_2]D^u_l D^d + [p^u_{h,2} - c(q_h) + g(q_h)p^d_2]D^u_h D^d, \tag{5}$$

where (p^u_1, p^d_1) denotes prices when one-quality level is offered, and $(p^u_{l,2}, p^u_{h,2}, p^d_2)$ denotes the prices when two-quality level is offered. For simplicity, we only use the subscript $(1, 2)$ when we need to differentiate between the one-quality and two-quality cases. Specifically, we suppress the subscript $(1, 2)$ whenever possible except Section 3 and the proofs in Appendix B. That is, (p^u, p^d) (or (p^u_l, p^u_h, p^d_2)) indicate prices with one-quality (or two-quality) level unless specified otherwise.

buyer with which a developer interacts. Thus, the total number of interactions by developers is not affected by the type of buyer with which they interact.

Note that we endogenize the platform's decision regarding whether to provide quality differentiation. As we will show, the platform prefers quality differentiation with high- and low-quality levels to buyers because it is more profitable than one-quality provision. Nevertheless, we analyze the case of one-quality provision in Section 2.1 as a benchmark.

Throughout the paper, we make the following assumptions.

Assumption 1. *The cost is increasing and convex in quality:* $c'(q_k) > 0$, $c''(q_k) > 0$.

This assumption suggests that it becomes increasingly costly to provide higher-quality service, which is a standard assumption.

Assumption 2. *The density functions for buyers and developers are increasing in type:* $(f^u)' \geq 0$ *and* $(f^d)' \geq 0$*, which implies that* $f^u(1) > 0$*, and the inverse hazard rate is non-increasing in buyer type:* $\frac{\partial \frac{1-F^u(\theta)}{f^u(\theta)}}{\partial \theta} \leq 0$ *and* $\frac{\partial \frac{1-F^d(\theta)}{f^d(\theta)}}{\partial \theta} \leq 0$.

Assumption 2 implies that we impose the standard monotone hazard rate condition on F. Additionally, it guarantees that as buyers attribute more value to the per usage benefit, there are more platform buyers than nonbuyers.

Timing of the game The timing of the game is set as follows.

0. Before the main game begins, the platform is assumed to provide one quality q_l to buyers at p^u, which are the basic (low) quality level and the price, respectively. The price for developers is set at p^d.

1. The platform decides whether to provide two levels of quality to buyers. If engaging in quality differentiation, it determines the price levels for buyers (p^u_l and p^u_h), given the exogenously given quality levels (basic low q_l and high q_h). It also optimally sets the price for developer p^d_2.

2. Buyers and developers make participation decisions.

The solution concept we use for this game is the perfect Bayesian Nash Equilibrium (PBE) for multiperiod games with observed action, which can be solved by backward induction. PBE consists of a sequentially rational strategy profile for all players and a set of consistent beliefs about buyer's and developers' valuations with respect to benefits from using the platform.

2.1 Model with the one-quality case

As a benchmark, we begin by considering a model that omits the practice of quality differentiation and then modify the model to allow differentiation in Section 2.2. The two-sided monopolist offers a single quality of access in this case. The buyer's utility is given by Equation (2) with $q_k = q_l$ and $p^u_{k,m} = p^u$. Note again that we suppress the subscript differentiating one-quality from two-quality case in Sections 2.1 and 2.2 for notational ease.

We first analyze the user's side (buyers and developers) to identify the equilibrium demand. The equilibrium demand functions are derived from the participation constraint:

$$\begin{aligned} D^u = Prob\,(U^u \geq 0) \Leftrightarrow D^u = 1 - F^u\left(\frac{p^u}{\mathcal{U} q_l}\right). \\ D^d = Prob\left(U^d \geq 0\right) \Leftrightarrow D^d = 1 - F^d\left(\frac{p^d}{\mathcal{D}}\right). \end{aligned} \tag{6}$$

Note that the equilibrium supply D^d is not affected by $g(q_l)$. This result comes from the assumption that the effect of quality, measured by $g(q_l)$, affects developers' revenues, namely, $\mathcal{D}\alpha^d_j$, and fees paid to the platform, namely, p^d, in the same manner. In most cases, the fees charged to developers are proportional to their revenues earned from the platform. For example, a video game platform takes a certain percentage of purchases made by buyers from games. High-quality buyers, such as those using *Xbox Series X*, are more likely to make additional purchases in games, especially physical editions of games, than are those with low-quality consoles, such as *Xbox Series S*, because the low-quality console, which has no optical drive, does not provide the physical edition of games; therefore, game developers can earn more revenues proportionally from high-quality provisions. That is, if the effect of quality improvement on the buyer's side on developer's revenues is measured by $g(q_l)$, that on the fees can be represented by $\delta g(q_l)$, where $\delta \in (0, 1)$. For simplicity, we normalize δ to one; however, even if δ is between zero and one, i.e., the effect of quality on developers' revenues is different from that on fees, the qualitative results would hold as long as the first effect dominates the second.

From the platform's profit maximization problem, as in Equation (4), we can solve for the optimal solutions for the prices on the buyer's and developer's sides as follows:[7]

$$\frac{p^u - c(q_l) + p^d}{p^u} = \frac{1}{\varepsilon^u}; \quad \frac{p^d - c(q_l) + p^u}{p^d} = \frac{g(q_l)}{\varepsilon^d}. \tag{7}$$

From Equation (7), we obtain the equilibrium condition that captures the trade-off between p^u and p^d as follows:

$$\frac{p^u}{\varepsilon^u} = \frac{p^d \times g(q_l)}{\varepsilon^d}. \tag{8}$$

Comparing Equation (8) to the equilibrium condition in the basic model of Rochet and Tirole (2003), we find a more interesting interaction between the effect of quality on the developer's side and

[7]The details are in Appendix A.1.

relative price sensitivity. Specifically, if the positive effect of quality improvement on the increase in developer's side revenue, measured by $g(q_l)$, is greater (less) than the relative price sensitivity between buyers and developers, measured by $\frac{\varepsilon^d}{\varepsilon^u}$, the platform charges a lower (higher) fee to developers than to buyers.

Proposition 1. *The optimal relative price for buyers to that for developers depends not only on the relative price sensitivity between the two sides but also on the external effect of quality improvement on the buyer's side on the developer's side. Specifically, if the positive effect of quality improvement on the developer's side is less than the relative price sensitivity between the two sides, then the platform charges a higher price for developers than for buyers.*

$$g(q_l) \leq (\geq) \frac{\varepsilon^d}{\varepsilon^u} \iff p^u \leq (\geq) p^d.$$

Furthermore, increasing quality gives the monopolist an incentive to increase prices on both sides.

In Proposition 1, we not only show how quality improvement affects the platform's pricing strategy on both sides in general but also show how the effect of quality affects the platform's incentive to charge relative prices for developers and buyers. This result is particularly important since we consider the quality measure, even without quality differentiation, as one additional channel for determining the optimal conditions for prices.

As mentioned above, Equation (8) is similar to the equilibrium of the two-sided market, as in Rochet and Tirole (2003). However, our equilibrium condition still differs from that in Rochet and Tirole (2003) insofar as we focus on a three-way trade-off that includes the exogenous quality variable in addition to a trade-off between prices on the two sides. Allowing the platform to provide differentiated quality tiers for the buyer side creates more opportunities for optimal business strategies. The choice of quality provision offers another level of flexibility for profit maximization, as it can increase the revenues generated by developers. Indeed, as we will show, the platform's optimal pricing decisions depend on its quality differentiation choices. Furthermore, as demonstrated in Proposition 1, even in the single-quality case, the quality of access on the buyer side impacts the price for developers.

2.2 Model with two qualities case

As mentioned above, the platform's basic low-quality level does not vary when a high-quality service is introduced, i.e., the basic quality level is given by q_l. Thus, the choice variables for the platform include p_l^u, p_h^u, and p_2^d.

We start by analyzing the user's side (buyers and developers) to identify the equilibrium demand.

First, the buyers have two choices for accessing the platform. They can join the platform through either low-quality access or high-quality access. Given the two types of quality, high and low, the number of participants with low-quality access is determined by the number of buyers who satisfy the following two conditions:

1. (IR constraint) The buyer's utility from low-quality access is greater than zero: $Pr(U_l^u \geq 0)$.
2. (IC constraint) Buyers for whom the utility derived from low-quality access exceeds that from high-quality access: $Pr(U_l^u \geq U_h^u)$.

The two conditions jointly determine the proportion of low-type buyers.

$$D_l^u = \Pr\left(\frac{p_h^u - p_l^u}{\mathcal{U}(q_h - q_l)} \geq \alpha^u \geq \frac{p_l^u}{\mathcal{U}q_l}\right) = F^u\left(\frac{p_h^u - p_l^u}{\mathcal{U}(q_h - q_l)}\right) - F^u\left(\frac{p_l^u}{\mathcal{U}q_l}\right), \tag{9}$$

where $D_l^u \equiv D^u(p_h^u, p_l^u, q_h, q_l)$. Similarly, the number of participants joining the high-quality service is given by the number of buyers who satisfy the following two conditions:

1. (IR constraint) The buyer's utility from the high-quality good is greater than zero: $Pr(U_h^u \geq 0)$.
2. (IC constraint) Buyers for whom the utility derived from the high-quality good exceeds that from the low-quality good: $Pr(U_h^u \geq U_l^u)$.

The IR condition is satisfied when the IC constraint of the high type and IR constraint of the low type hold. Thus, the proportion of high-type buyers is given by the following:

$$D_h^u = \Pr\left(\alpha^u \geq \frac{p_h^u - p_l^u}{\mathcal{U}(q_h - q_l)}\right) = 1 - F^u\left(\frac{p_h^u - p_l^u}{\mathcal{U}(q_h - q_l)}\right), \tag{10}$$

where $D_h^u \equiv D^u(p_h^u, p_l^u, q_h, q_l)$. Given the utility function for buyers, the total number of buyers joining the platform is given by the following:

$$D^u = \Pr(U_l^u \geq 0) = \Pr(\mathcal{U}\alpha^u q_l \geq p_l^u) = \Pr(\alpha^u \geq \frac{p_l^u}{\mathcal{U}q_l}) = 1 - F^u\left(\frac{p_l^u}{\mathcal{U}q_l}\right), \tag{11}$$

where $D^u \equiv D(p_l^u, q_l)$. Equation (11) shows how the number of participants on the buyer's side depends only on the price and quality of the low-quality good. Although there are network externalities in the total utility derived from the platform or the gross transaction utility, the per unit transaction demand is not dependent on the participation rate on the other side.[8] This condition is necessary because the participation constraint (IR constraint) for high-quality buyers is slack. This means that

[8] This setup has one restriction that we need to impose, which is that the proportion of low-type buyers has to be nonnegative: $D_l^u \geq 0$.

the participation of low-quality buyers guarantees the participation of high-type buyers. In other words, the buyers on the margin of joining the platform are low-quality buyers.

The total number of developers who join the platform is given by the following:

$$D^d = \Pr(U^d \geq 0) = 1 - F^d\left(\frac{p_2^d}{\mathcal{D}}\right), \tag{12}$$

where $D^d \equiv D^d(p_2^d)$. Given the total number of buyers and developers, the equilibrium level of participation is the following:

$$\begin{aligned} &D^u = D(p_l^u, q_l) = 1 - F^u\left(\frac{p_l^u}{\mathcal{U}q_l}\right); \quad D^d = D^d(p_2^d) = 1 - F^d\left(\frac{p_2^d}{\mathcal{D}}\right). \\ &D_l^u = F^u\left(\frac{p_h^u - p_l^u}{\mathcal{U}(q_h - q_l)}\right) - F^u\left(\frac{p_l^u}{\mathcal{U}q_l}\right); \quad D_h^u = D^u(p_h^u, p_l^u, q_h, q_l) = 1 - F^u\left(\frac{p_h^u - p_l^u}{\mathcal{U}(q_h - q_l)}\right). \end{aligned} \tag{13}$$

Given quality differentiation, the monopoly problem given by Equation (5) can be rearranged as follows:

$$\max_{p_2^d, p_l^u, p_h^u} \Pi = (\pi_l D^u + \pi_{h-l} D_h^u) D^d, \tag{14}$$

where $\pi_l \equiv p_l^u + g(q_l)p_2^d - c(q_l)$ and $\pi_{h-l} \equiv (p_h^u - p_l^u) + (g(q_h) - g(q_l))p_2^d - (c(q_h) - c(q_l))$. Note that π_l and π_{h-l} represent the platform's per transaction profit from low-quality access and from high-quality access, respectively. The following is the breakdown of the equilibrium prices and quality for buyers and developers.[9]

2.2.1 Price of low-quality access on the buyer's side

The price of low-quality access on the buyer's side can be obtained as follows:

$$\pi_l = \frac{D^u}{(-D^u)'_{p_l^u}} = \frac{p_l^u}{\varepsilon^u}, \tag{15}$$

where $(D^u)'_{p_l^u}$ means that $\frac{\partial D^u}{\partial p_l^u}$, and the price elasticity of demand is represented by ε^u. The optimal price for low-quality access is determined at the level at which the per transaction profit from low-quality access is equal to an additional markup obtained from low-quality buyers, i.e., $\frac{p_l^u}{\varepsilon^u}$.

2.2.2 Price of high-quality access on the buyer's side

The price of high-quality access on the buyer's side can be obtained as follows:

$$\pi_{h-l} = \frac{D_h^u}{(-D_h^u)'_{p_h^u}} = \frac{p_h^u}{\varepsilon_h^u}, \tag{16}$$

[9] The details are provided in Appendix A.2.

where $(D^u_h)'_{p^u_h}$ means that $\frac{\partial D^u_h}{\partial p^u_h}$, and the price elasticity of demand for high-quality access is represented by ε^u_h. The optimal price for high-quality access is determined at the level at which the per transaction profit from high-quality access is equal to an additional markup obtained from high-quality buyers, i.e., $\frac{p^u_h}{\varepsilon^u_h}$.

2.2.3 Price for developers

We now turn our attention to the price for developers. For ease of notation, let the ratio of consumers on high-quality access be $\frac{D^u_h}{D^u} \equiv \lambda_h$.

$$\pi_l + \pi_{h-l}\lambda_h = \frac{p^d}{\varepsilon^d}\left\{g(q_l) + [g(q_h) - g(q_l)]\lambda_h\right\}. \tag{17}$$

Equation (17) implies that the optimal price charged to developers is set at the level at which the average per transaction profit from buyers is equal to an additional markup obtained from developers, i.e., $\frac{p^d}{\varepsilon^d}\left\{g(q_l) + [g(q_h) - g(q_l)]\lambda_h\right\}$. That is, the optimal p^d is set by optimally balancing the marginal revenues earned from two different sides: buyers and developers.[10]

Equations (15), (16), and (17) give us the following:

$$\frac{p^u_l}{\varepsilon^u} + \frac{p^u_h}{\varepsilon^u_h}\lambda_h = \frac{p^d}{\varepsilon^d}\left\{g(q_l) + [g(q_h) - g(q_l)]\lambda_h\right\}. \tag{18}$$

3 Equilibrium Results

We derive several important implications from the model by showing how quality differentiation determined by the multi-sided platform affects its optimal use of the interaction between two sides, such as cross-subsidization.

3.1 Effects of quality differentiation on the platform and two sides

Before we proceed, we first show that the platform has an incentive to engage in quality differentiation in the first place: we find that quality differentiation leads to greater profit for the platform, as in Proposition 2. The detailed proof is provided in Appendix B.

Proposition 2. *The platform strictly prefers to charge different prices by quality on the buyer's side.*

Proposition 2 implies that a platform that provides only one type of quality is able to obtain more profit if it slightly differentiates product quality. Even a minor quality improvement with a small price

[10] Note that we have normalized the cost on the developer's side to zero, so $c^d = 0$.

increase can increase the platform's profit as long as it continues to provide differentiated products, such as low- and high-quality products. Thus, quality differentiation permits the platform to earn more profit by implementing premium service in addition to basic service, which not only expands the size of the total buyer market but also extracts more rents from relatively high-quality buyers.

Next, we focus on the multi-sided platform case with the provision of two quality levels to investigate how quality differentiation on the buyer's side affects the price for developers, the price for buyers, and the interaction between the two sides.

We first find that the price of the low-quality product for buyers and the fee for developers move in opposite directions as the platform differentiates the quality tier for buyers by introducing a high-quality product. Lemma 1 summarizes this finding.

Lemma 1. *If the monopolist introduces a high-quality product, the price of the low-quality product on the buyer's side and the fee charged on the developer's side move in the opposite direction.*

Unlike in the one-sided case, the profit-maximizing strategy of a two-sided firm is modified by re-evaluating the relative prices on the two sides. Specifically, quality differentiation, which affects both the buyer's and developer's sides' pricing strategies, allows the platform to cross-subsidize the two sides by having the price on the two sides move in the opposite direction to maximize the gain from quality differentiation.

Additionally, we find that cross-market externalities arises from quality differentiation on the buyer's side. That is, if the platform introduces high-quality access on the buyer's side, such quality differentiation generates externalities on the developer's side's pricing. Whether the external effects of quality differentiation increase or decrease the price for developers depends on the relative size of the average profit increase on the two sides. Proposition 3 summarizes this finding.

Proposition 3. *Upon quality differentiation on the buyer's side, the quality weighted price on the developer's side weakly increases if and only if the platform can extract more rent from the developer's side than from the buyer's side by providing higher quality, i.e.,*

$$p_1^d \leq p_2^d \iff \frac{\pi_{h-l}}{\pi_l} < \frac{[g(q_h) - g(q_l)]}{g(q_l)}. \tag{19}$$

In Equation (19), $\frac{\pi_{h-l}}{\pi_l}$ indicates the platform's normalized profit for high-quality access, where normalization is made by the base low-quality profit, and $\frac{[g(q_h)-g(q_l)]}{g(q_l)}$ is the relative extra rent that the platform can extract from the developer's side by providing quality differentiation on the buyer's side compared to one-quality (low-quality) case. The inequality on the right-hand side compares how much rent the platform can extract by quality differentiation from the developer's side with extra rent from the buyer's side. Thus, Proposition 3 implies that it is optimal for the platform to charge a higher price for developers in the two-quality case than in the one-quality case if it can extract more rents from the developer's side than from the buyer's side.[11]

For example, when a platform releases a high-quality game console in addition to a basic-quality console, such as *Xbox Series X* (high-quality) and *Xbox Series S* (basic low-quality), it can attract more buyers, and more buyers, in turn, attract more game developers. Thus, quality differentiation on the buyer's side increases the profit not only on that side but also on the other side, i.e., the developer's side, via cross-side network externalities. Proposition 3 states that if network externalities are sufficiently large such that the profit-increasing effects are greater on the other side than on the side involving differentiation, the platform optimally charges a higher price for developers upon quality differentiation, even if there is no quality improvement on the developer's side.

Combining Lemma 1 with Proposition 3, we further find that the relative price of the basic, low-quality product for buyers in the one-quality provision (i.e., p_1^u) case to that for the basic, low-quality, product in the two-quality provision case (i.e., $p_{l,2}^u$) also depends on which side gains more from quality differentiation. That is, the platform weakly lowers the price for the basic-quality product under quality differentiation if it earns more normalized profit for high-quality access from the developer's side than from the buyer's side. Thus, the introduction of a high-quality product on one side alters pricing dynamics between the two sides via network effects.

Following the two-sided market literature (e.g., Rochet and Tirole, 2003, 2006), a standard "see-saw principle" intuition can be seen here: that is, when quality differentiation and differential prices allow the platform to extract the buyer's side surplus more effectively, it leads to platform valuing the transaction volume and buyer participation more, and so it optimally decreases the developer's side price upon quality differentiation. In the special case such as $g(q_k) = 1\ \forall q_k$, where $k \in \{l, h\}$, i.e., the case where quality access on the buyer's side does not affect the utility of developers, it would be optimal to decrease the developer price in the case of quality differentiation, as predicted in the standard "see-saw principle."

Corollary 1. *If $g(q_k) = 1\ \forall q_k$, where $k \in \{l, h\}$, then quality differentiation decreases the price on the developer's side.*

However, in a broader and more nuanced scenario, quality improvement affects both the buyer and the developer sides. Thus, the effect of quality differentiation on price depends on the comparative

[11] To better describe the intuition behind Proposition 3 in a clearer way, we add a numerical example, which shows the results, in Appendix A.4.

value improvement for both developers and buyers (as discussed in Proposition 3).[12]

3.2 The combined effects of openness and quality differentiation

Thus far, we have considered the platform to be a multi-sided firm and have not investigated how the effects of quality differentiation for a multi-sided platform are different from those for a one-sided firm. By comparing the effects of quality differentiation on a one-sided firm to those on a two- or multi-sided firm, we can see how serving more sides as the platform, which we call openness in the paper, is affected differently by quality differentiation through exploiting the interactions between different sides. In this regard, we generalize the model further to the case where the monopolist also optimizes the quality level in addition to the price level.

If the monopolist serves only one side of the market, say the buyer's side, its profit function is given by the following:

$$\max_{p^u_{os}, q_l} \Pi_{\text{one-sided monopolist}} = [p^u_{os} - c(q_l)]D^u, \tag{20}$$

where the subscript os indicates one-sided market (in the case of one-quality), which is introduced to distinguish it from p^u, which indicates the two-sided platform case.

We find that the monopolistic firm provides higher quality to buyers for each dollar that they pay when it serves more sides of the market. As the monopolist opens more sides to serve, for example, such openness increases the quality per dollar offered to buyers. Mathematically, the quality per dollar is denoted as $\frac{q_l}{p^u}$. This finding is summarized in Proposition 4.

Proposition 4. *As a firm serves more sides of the market, it provides higher quality per dollar offered to buyers:* $\frac{q_l}{p^u_{os}} \leq \frac{q_l}{p^u}$.

Intuitively, the monopolist serving multiple sides has more incentive to offer a better-quality price ratio to buyers because it now obtains more profit from the developer's side. If the two-sided firm offers a better-quality price menu to buyers, it attracts more of them. When it serves the developer's side at the same time, more demand from the buyer's side means that the developers on the other side earn more revenue. The platform can extract additional revenues on the developer's side, which incentivizes it to offer a better-quality price ratio to buyers, ultimately attracting more buyers than a one-sided platform.

Moreover, both traditional one-sided firms and two-sided platforms can offer differentiated quality tiers for buyers. In this case, the logic in Proposition 4 implies that servicing both sides makes the

[12]Note that in the general case of quality differentiation, we will have mark-ups on both sides and thus the 'see-saw' principle is not applicable when $g(q_k) \neq 1$.

platform more willing to offer a better deal to low-type members by providing high quality per dollar at the basic level than a one-sided firm. For example, the average quality of services provided by taxi companies, as traditional one-sided firms, is known to be lower than that provided by ride-sharing platforms, such as Uber and Lyft. As Liu et al. (2019) show, taxi drivers are more likely to detour with non-local customers, which results in longer travel time. Such empirical evidence supports our finding that the platform's basic quality provision is better than that of a one-sided firm.

The reason the platform provides a higher quality for basic service is that it is able to exploit such quality improvement on one side to encourage more participation from the other side, thereby raising its profits. The underlying incentives for platforms to engage in higher service provision than a one-sided firm arise from this cross-subsidization motive. This finding is summarized in Corollary 2.

Corollary 2. *The platform in a two- or multi-sided market is more likely to offer a higher-quality basic service or product than a one-sided firm.*

4 Implications for the video game industry

The video game industry has multiple buyers who interact through a gaming platform. In particular, there are three main types of gaming platform: (i) open-source platforms accessed through a PC such as social platforms, e.g., Facebook; (ii) game consoles; and (iii) mobile gaming. For application of this model, a game console is the best fit as the only utility derived from the platform arises from playing games on it, whereas buyers of PC or mobile game platforms derive other benefits apart from using the platform as a gaming platform. Moreover, the game consoles need to have both buyers and developers on the platform to make a profit. Thus, the market generates positive cross-network externalities as the number of buyers using the console has a positive effect on games that are developed for the platform and vice versa.[13] Using the game console industry as an example, we summarize several managerial insights derived from our findings.

4.1 The interaction between quality access on the buyer's side and price ratio between the two sides.

As discussed above, the two sides in the game industry, i.e., the game buyers and developers, are interrelated by cross-network effects. In other words, the value derived from the platform for each

[13] For more details on platform dynamics in this market, refer to Shankar and Bayus (2003), Venkatraman and Lee (2004), Clements and Ohashi (2005), Prieger and Hu (2006), Zhu and Iansiti (2007), Nair (2007), Corts and Lederman (2008), Derdenger (2008, 2011), Lee (2013), Binken and Stremersch (2009), Liu (2010), Dubé et al. (2010), Chao and Derdenger (2011).

side depends on the size of the buyers on the other side. Under these conditions, does the quality of the platform (e.g., graphics, display of PlayStation) on one side (buyer's side) affect the other side (developer's side)? If so, does the platform internalize this effect at the optimal prices? The first main finding of our paper answers these questions. By extending Rochet and Tirole (2003) to endogenize the impact of the quality of the game console for the buyer's side on the profit made by developers, we find that the optimal two-sided pricing in the extended model is affected by the quality of the console. In particular, we find that the quality of the game console changes the pricing ratio between the two sides.

Recall that quality differentiation for buyers affects both the buyer's side because buyers are willing to pay a higher price for a better-quality console and the developer's side via revenues generated by game developers. The former is the effect of price differentiation on the profit margin incurred from the buyer's side, which is known in the literature. The latter is the effect of the quality of the game console on the cross-side network externalities arising from the multi-sided nature: the level of quality of the game console impacts the time spent playing games by the buyer, which in turn impacts the profit earned by the developers. Proposition 1 states that if the positive effect of quality improvement on the developer's side is greater than the relative price sensitivities between the two sides, then the platform charges a higher price for developers than for buyers.

These findings also match the pricing decisions observed in this market. According to sector analysts, Microsoft sold its *Xbox 360* console at a price of at least $125, which was under its marginal cost. Sony's PlayStation 2 console, which costs $299 at the time of its release and was also sold at a loss.[14] Such examples could indicate that the positive effects of the buyer's side's quality improvement on the developer's side are sufficiently large; thus, the platform extracts more rents from the developer's side via higher fees, whereas it lowers the price for buyers.[15] This finding has managerial implications for the quality of the product for one side of the market: the access quality for buyers not only affects the price on the buyer's side but also influences the price on the developer's side through cross-side network externalities.

Specifically, our theoretical prediction recommends that in order to maximize revenue, it is optimal for the game console platform to change the price/royalties charged on the developer's side when a new quality game console is introduced to buyers. This cross-side network externality drives the change in royalties paid by developers when a new quality game console is introduced. In this regard, Lemma

[14]See, for example, Herald News Service 30 March (2000, p. 56) and "Sony losing over US 300 on each 20GB PS3", http://www.gamesindustry.biz.

[15]Note that we interpret that below-cost pricing for the buyer's side indicates that the platform generates profit from the other side by means of a higher price for developers.

1 and Proposition 3 show that allowing quality differentiation makes the platform consider relative profits between two sides when setting an optimal price. Specifically, if quality differentiation on the buyer's side leads to greater average additional profit from the developer's side, then the platform lowers the price for the basic-quality product even if there is no quality degradation, whereas it raises the price for developers. Thus, quality differentiation can be one of the market aspects that a platform in a two-sided market takes into account to exploit cross-market externalities.

Note that we have not taken into account the issue of compatibility in quality differentiation. For example, while a high-quality console such as the *PS4 Pro* is compatible with any games offering higher resolution, a lower-quality console such as the *PS4 Slim* does not support games with 4K resolution. Considering this aspect, quality differentiation may dissuade some buyers who are concerned about compatibility from upgrading to higher-quality options, and/or it may necessitate developers to incur additional investment costs to launch games compatible with high-quality access. Although these dynamics impact the equilibrium of quality differentiation, we abstract away such aspects for now to focus on the first order importance of introducing additional quality access in the market.

4.2 The optimal price for buyers of game consoles with two quality tiers and its openness

Again, our model focuses on the case in which the quality of the game console generates positive cross-side network externalities. Indeed, the quality differentiation of the game console (e.g., PlayStation *Pro* vs. *Slim*) can improve the gaming experience for buyers, thereby increasing the revenues for developers on the other side. Thus, unlike one-sided firms, which determine the prices for game consoles with two different quality tiers on the basis of price elasticity on the buyer's side, the two-sided platform has an additional effect to consider, which is the effect of quality differentiation on cross-side network effects. Specifically, Proposition 4 shows that the price charged for low-quality access (e.g., PlayStation *Slim*) by a gaming platform would be lower for a two-sided platform than for a one-sided firm. In terms of managerial implications, this means that managers need to account for the two-sided nature of the interaction when deciding prices for different quality tiers of game consoles.

For example, Sony decreased the price for PlayStation 2 (PS2) when a new generation with better-quality features, PlayStation 3, was launched.[16] The lower price for PS2 increased the market size for buyers, which had positive effects on the developer's revenue. In this case, our model suggests that one of the reasons for decreasing the price of PS2 on the buyer's side could be to extract a higher profit from the developer's side by increasing the royalty charged to the game developers.

[16] See Table 1 in Davidovici-Nora et al. (2012) for details.

Finally, it is worth noting that the basic intuition holds for other platforms that have two sides, such as mobile gaming and social media platforms such as Facebook. For example, similar to our model, both Facebook and mobile app stores such as iOS charge fees to gaming apps for using their platforms. Therefore, we believe that the model can be extended to these markets as long as the platform charges fees to developers.

5 Conclusion

In this paper, we use game theory to show that in addition to price, platforms can also strategically use the quality of platform quality to determine the optimal price. We further show how differentiating based on quality access leads to new insights in the platform setting. For example, Amazon offers buyers two types of quality access: basic (low-quality) access is free, while premium access (*Prime* membership, which is high-quality) is the paid service. In the video game industry, Sony launched PlayStation Pro (high-quality) and Slim (low-quality). In this setting, we show how quality access impacts the relative price on the two sides (e.g., game buyers and developers in the game console case). Furthermore, we find that providing different quality access on the buyer's side also impacts the other side (i.e., affects the fee charged to developers in the game console example and sellers in the Amazon example).

Overall, this paper introduces a simple model that sheds light on the impact of platform quality differentiation on cross-side network externalities. Our findings suggest one plausible business strategy for the platform, that is, how quality differentiation implemented by the platform can be used as an optimal business strategy. It would be interesting for future research to examine a dynamic setting in which buyers await a high-quality platform or buy it in the current period. Additionally, researchers could investigate how competition in the platform market alters our results. In this regard, a model with an asymmetric setup in the competitive market structure faced by the platform could determine the extent to which such asymmetry affects the platform's optimal business strategy. Lastly, a possible interesting extension of this model would be to make the quality level and its impact on developers as endogenous choice variables for the platform.

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Appendix

A Further Discussion

A.1 Details of the equilibrium price and quality equations for one-quality case

The profit function under one quality provision case can be simply obtained by assuming that π_{h-l} is zero. Thus, in the one-quality case we will solve the case where the platform will offer only the low-quality access, i.e., (p^u, q_l).

If one type of quality is offered, the platform solves its profit maximization problem with respect to p^u and p^d, as given below:

$$\max_{p^d,p^u} \Pi = \pi_l D^u D^d.$$

$$\Rightarrow \max_{p^d,p^u} \Pi = [p^u - c(q_l) + g(q_l)p^d] D^u D^d. \tag{21}$$

The first order condition with respect to p^u is given as follows.

$$\begin{aligned} & [p^u + g(q_l)p^d - c(q_l)](D^u)' D^d + D^u D^d = 0 \\ \Leftrightarrow \quad & [p^u + g(q_l)p^d - c(q_l)] = \frac{D^u}{(-D^u)'} = \frac{p^u}{\varepsilon^u}, \end{aligned} \tag{22}$$

where $(D^u)'$ means that $\frac{\partial D^u}{\partial p^u}$.

The first order condition with respect to p^d is given as follows.

$$\begin{aligned} & [p^u + g(q_l)p^d - c(q_l)](D^d)' D^u + g(q_l) D^d D^u = 0. \\ \Leftrightarrow \quad & [p^u + g(q_l)p^d - c(q_l)] = \frac{g(q_l)D^d}{(-D^d)'} = \frac{g(q_l)p^d}{\varepsilon^d}, \end{aligned} \tag{23}$$

where $(D^d)'$ means that $\frac{\partial D^d}{\partial p^d}$.

Using Equations (22) and (23), we obtain the following equation.

$$\frac{p^u}{\varepsilon^u} = \frac{g(q_l)p^d}{\varepsilon^d}.$$

A.2 Details of the equilibrium price and quality equations for two-quality case

If the platform differentiates its offered quality tiers, the platform solves its profit maximization problem with respect to p_l^u, p_h^u, and p_2^d, as in Equation (5). We suppress the subscript for one-quality versus two-quality case in the buyer's side for notational ease. First, the first order condition with respect to p_l^u is given as follows.

$$\begin{aligned} & (D^u - D_h^u) D^d + \pi_l (D^u)'_{p_l^u} D^d + \pi_{h-l} (D_h^u)'_{p_l^u} D^d \\ = \quad & D_l^u + \pi_l (D^u)'_{p_l^u} + \pi_{h-l} (D_h^u)'_{p_l^u} = 0, \end{aligned} \tag{24}$$

where $(D_h^u)'_{p_l^u}$ means that $\frac{\partial D_h^u}{\partial p_l^u}$. Additionally, $[p_l^u + g(q_l)p^d - c(q_l)] \equiv \pi_l$ and $[(p_h^u - p_l^u) + (g(q_h) - g(q_l))p^d - (c(q_h) - c(q_l))] \equiv \pi_{h-l}$. We also denote $\frac{D_h^u}{D^u} \equiv \lambda_h$.

The first order condition with respect to p_h^u is obtained as follows.

$$\pi_{h-l}(D_h^u)'_{p_h^u}D^d + D_h^u D^d = 0 \quad \Leftrightarrow \quad \pi_{h-l} = \frac{D_h^u}{(-D_h^u)'_{p_h^u}} = \frac{p_h^u}{\varepsilon_h^u}, \tag{25}$$

where $(D_h^u)'_{p_h^u}$ means that $\frac{\partial D_h^u}{\partial p_h^u}$. By using Equation (25), we simplify Equation (24) as follows.

$$\begin{aligned} & D_l^u + \pi_l (D^u)'_{p_l^u} + \left[-\frac{D_h^u}{(D_h^u)'_{p_h^u}} \right] (D_h^u)'_{p_l^u} \\ = \quad & D_l^u + \pi_l (D^u)'_{p_l^u} + \left[\frac{D_h^u}{(-D_h^u)'_{p_h^u}} \right] (-D_h^u)'_{p_h^u} \\ = \quad & D_l^u + \pi_l (D^u)'_{p_l^u} + D_h^u = 0 \quad \Leftrightarrow \quad \pi_l = \frac{D^u}{(-D^u)'_{p_l^u}} = \frac{p_l^u}{\varepsilon^u}, \end{aligned} \tag{26}$$

where the second equality comes from $(D_h^u)'_{p_l^u} = -(D_h^u)'_{p_h^u}$,

Lastly, the first order condition for p_2^d is given as follows.

$$(\pi_l D^u + \pi_{h-l} D_h^u)(D^d)'_{p_2^d} + g(q_l) D^u D^d + [g(q_h) - g(q_l)] D_h^u D^d = 0. \tag{27}$$

By using the definition of $\lambda_h \equiv \frac{D_h^u}{D^u}$, we can simplify Equation (27) as follows.

$$\begin{aligned} & (\pi_l + \pi_{h-l}\lambda_h)(D^d)'_{p_2^d} + \{g(q_l) + [g(q_h) - g(q_l)]\lambda_h\} D^d = 0. \\ \Leftrightarrow \quad & \pi_l + \pi_{h-l}\lambda_h = \frac{p_2^d}{\varepsilon^d} \{g(q_l) + [g(q_h) - g(q_l)]\lambda_h\}. \end{aligned} \tag{28}$$

Combining equilibrium conditions for all prices in Equations (25), (26) and (28), we obtain the following:

$$\frac{p_l^u}{\varepsilon^u} + \frac{p_h^u}{\varepsilon_h^u}\lambda_h = \frac{p_2^d}{\varepsilon^d} \{g(q_l) + [g(q_h) - g(q_l)]\lambda_h\}. \tag{29}$$

A.3 Proof of equivalence of the main model with the fixed fee case

Here, we show that the setup with per transaction fee is analogous to the case where both fixed and transaction fees are charged on the user side.[17] Again, we are suppressing the $m \in \{1, 2\}$ subscript here for convenience.

Without loss of generality, let us assume that the buyers' side charged a per transaction fee and a fixed fee, which implies that buyers have the following utility:

$$U^u = (\mathcal{U}\alpha^u q_l - p^u)N^d - P^u, \tag{30}$$

where P^u is the fixed fee charged to the buyers. The seller has the following utility:

[17] This change does not affect the model as long as there are no fixed benefits for the users.

$$U^d = (\mathcal{D}\alpha^d - p^d)N^u. \tag{31}$$

We first analyze the users' side (buyers and sellers) to identify the equilibrium demand. The equilibrium demand functions are derived from the participation constraint:

$$\begin{aligned} D^u = Prob\left(U^u \geq 0\right) \Leftrightarrow D^u = 1 - F^u\left(\frac{p^u + \frac{P^u}{N^d}}{\mathcal{U}q_l}\right). \\ D^d = Prob\left(U^d \geq 0\right) \Leftrightarrow D^d = 1 - F^d\left(\frac{p^d}{\mathcal{D}}\right). \end{aligned} \tag{32}$$

The monopoly problem can be written as follows:

$$\max_{p^d, p^u, q_l} \Pi = [p^u + p^d - c(q_l)]D^u D^d + P^u D^u. \tag{33}$$

Now, we modify the above case to show the equivalence. Let $p^u_{new} = p^u + \frac{P^u}{N^d}$ be the per transaction fee on the buyers' side and let the fixed fee be zero. Buyers have the following utility:

$$\begin{aligned} U^u &= (\mathcal{U}\alpha^u q_l - p^u)N^d - P^u \\ &= (\mathcal{U}\alpha^u q_l - p^u - \frac{P^u}{N^d})N^d \quad = (\mathcal{U}\alpha^u q_l - p^u_{new})N^d. \end{aligned} \tag{34}$$

Thus, the utility is the same in the case of (i) usage fee p^u as the fixed fee, and (ii) usage fee p^u_{new}. The seller has the following utility:

$$U^d = (\mathcal{D}\alpha^d - p^d)N^u. \tag{35}$$

We first analyze the users' side (buyers and sellers) to identify the equilibrium demand. The equilibrium demand functions are derived from the participation constraint:

$$\begin{aligned} D^u = Prob\left(U^u \geq 0\right) \Leftrightarrow D^u = 1 - F^u\left(\frac{p^u_{new}}{\mathcal{U}q_l}\right). \\ D^d = Prob\left(U^d \geq 0\right) \Leftrightarrow D^d = 1 - F^d\left(\frac{p^d}{\mathcal{D}}\right). \end{aligned} \tag{36}$$

The monopoly problem can be written as follows:

$$\begin{aligned} \max_{p^d, p^u, q_l} \Pi &= [p^u + p^d - c(q_l)]D^u D^d + P^u D^u \\ &= [p^u + p^d - c(q_l) + \frac{P^u}{D^d}]D^u D^d \\ &= [p^u_{new} + p^d - c(q_l)]D^u D^d. \end{aligned}$$

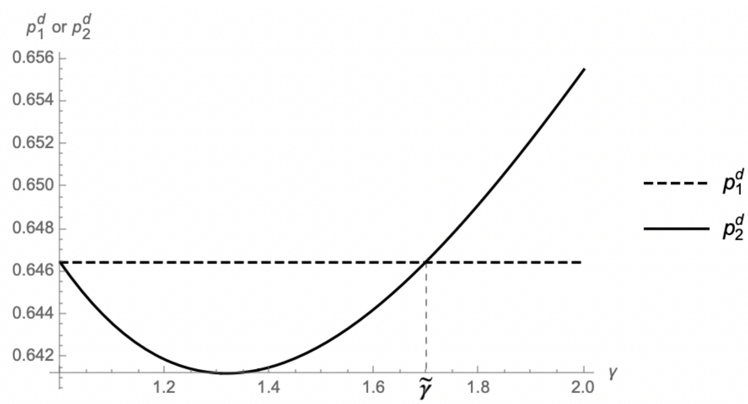
Thus, as shown above, the profit function for the platform and the users utilities are the same in these two cases. □

A.4 Numerical Exercise for Proposition 3

Proposition 3 states that it is optimal that the platform charges higher price for developers under the two-quality case than under the one-quality case (i.e., $p_1^d \leq p_2^d$) if it can extract more rents from the developer's side than the buyer's side (i.e., $\frac{\pi_{h-l}}{\pi_l} < \frac{[g(q_h)-g(q_l)]}{g(q_l)}$). To describe the intuition in a clearer way, we provide a numerical example underlying the finding.

Given that F^u and F^d are uniform distributions on $[0,1]$, $c(q_l) = q_l^2$, $g(q_l) = \sqrt{q_l}$, it is easy to show that $p_1^d = \frac{\sqrt{q}(q-\mathcal{U})+2\mathcal{D}}{3}$. Letting $q_h = \gamma q_l$ where $\gamma > 1$, we have the equilibrium p_2^d as a function of q_l, γ, $\mathcal{U}$, and $\mathcal{D}$.[18]

Under our numerical example, the term $\frac{[g(q_h)-g(q_l)]}{g(q_l)}$ in Equation (19) becomes $\sqrt{\gamma}-1$, implying that the condition, $\frac{\pi_{h-l}}{\pi_l} < \frac{[g(q_h)-g(q_l)]}{g(q_l)}$ becomes easier to be satisfied as γ increases. In other words, as γ increases, meaning that the platform can extract more rent from the developer's side, p_2^d is set at a higher level than p_1^d as shown in the figure below.



Specifically, we can show that both $\frac{\pi_{h-l}}{\pi_l}$ and $\frac{[g(q_h)-g(q_l)]}{g(q_l)} = \sqrt{\gamma}-1$ increase in γ; however, if γ is larger than a certain threshold (say, $\tilde{\gamma}$), we have $\frac{\pi_{h-l}}{\pi_l} < \frac{[g(q_h)-g(q_l)]}{g(q_l)}$. As shown in the figure above, we can also show that $p_1^d \leq p_2^d$ under the same condition on γ.

Additionally, the numerical example also shows that the relative rent extraction from both sides given high-quality access is dependent on the difference between the two quality levels (q_l and q_h). As we can see in the figure above, the change in extra rent and the price on the developer's side arising from quality differentiation depends on γ. This implies that as the level of the high-quality access increases it increases the rent extracted from the developer's side, and after a certain threshold passes, it leads to an increase in price on the developer's side. Thus, this example sheds light on possible implications of differences in quality (i.e., $q_h - q_l$) on change in price due to quality differentiation.

[18]We have a closed form solution to the equilibrium p_2^d, which is omitted here and can be provided upon request.

B Proofs of Propositions

Proof of Proposition 1. Let us prove the first case. Assume $g(q_l) \leq \frac{\varepsilon^d}{\varepsilon^u}$. We use the above assumption and the equilibrium Equation (8) stated below:

$$\frac{p_1^u}{\varepsilon^u} = \frac{p_1^d \times g(q_l)}{\varepsilon^d}.$$

The assumption and above equation can hold if and only if $p_1^u \leq p_1^d$. Hence, the proof for the other case is analogous to that above.

Now, to see the impact of the quality effect on prices, we use the equilibrium conditions reproduced below:

$$\begin{aligned} F_1 &\equiv p_1^u - c(q_l) + p_1^d - \frac{p_1^u}{\varepsilon^u} = 0; \\ F_2 &\equiv p_1^d - c(q_l) + p_1^u - \frac{g(q_l)p_1^d}{\varepsilon^d} = 0. \end{aligned} \tag{37}$$

To obtain the effect of q_l on p_1^d, we apply the implicit function theorem to F_1 and find the following:

$$\frac{dp_1^d}{dq_l} = -\frac{-c'(q_l)}{1} > 0.$$

To obtain the effect of q_l on p^u, we apply the implicit function theorem to F_2 and find the following:

$$\frac{dp^u}{dq_l} = -\frac{-c'(q_l) - \frac{g'(q_l)p_1^d}{\varepsilon^d}}{1} > 0.$$

Thus, increasing quality has a positive impact on both developers' and buyers' side prices. □

Proof of Proposition 2. Let $(\bar{p}_1^u, \bar{p}_1^d, \bar{q}_l)$ be the profit maximization variable for the single-quality case. We prove whether the platform wants to set a nonzero demand for high-quality products at this price and quality level. The demand for high-quality products will be zero if

$$\frac{p_{h,2}^u - p_1^u}{\mathcal{U}(q_h - q_l)} = 1 \ \Leftrightarrow \ p_{h,2}^u - p_1^u = \mathcal{U}(q_h - q_l). \tag{38}$$

We choose $(p_{h,2}^{u*}, q_h^*)$ such that Equation (38) is satisfied and then determine whether the first order condition on $(p_{h,2}^u, q_h)$ shows that the platform will attempt to increase demand for the high-quality product above zero. The first order condition with respect to $p_{h,2}^u$ at $(\bar{p}_1^u, \bar{q}_l, \bar{p}_1^d, p_{h,2}^{u*}, q_h^*)$ is given as follows:

$$
\begin{aligned}
\Phi^{p^u_{h,2}} &= D^d \left\{ \{[p^{u*}_{h,2} - c(q^*_h)] - [\bar{p}^u_1 - c(\bar{q}_l)]\}(D^u_h)'_{p^u_{h,2}} + D^u_h \right\}. \\
&= D^d \left\{ \left\{ [p^{u*}_{h,2} - c(q^*_h)] - [\bar{p}^u_1 - c(\bar{q}_l)] \right\} (-f^u(1)) \right\} \qquad \text{as } \frac{p^u_{h,2} - \bar{p}^u_1}{\mathcal{U}(q_h - \bar{q}_l)} = 1. \\
&= D^d \left\{ \{\mathcal{U}(q^*_h - \bar{q}) - [c(q^*_h) - c(\bar{q}_l)]\} (-f^u(1)) \right\}. \\
&\leq 0 \quad \text{if } f^u(1) \neq 0 \text{ and } c'(\bar{q}_l) < \mathcal{U}.
\end{aligned} \tag{39}
$$

In a similar manner, we can prove that at q^*_h, the first order condition is greater than zero. Thus, the platform will decrease $p^u_{h,2}$ and increase q_h such that $D^u_h \neq 0$. Therefore, we see that the profit increases when offering two product qualities as long as $f^u(1) \neq 0$ and $c'(\bar{q}_l) < \mathcal{U}$. $c'(\bar{q}_l) < \mathcal{U}$ is true, as $c'(\bar{q}_l) < \mathcal{U}$ implies that $\frac{\bar{p}^u_1}{B\bar{q}_l} < 1$, which holds for nonzero demand. □

Proof of Lemma 1. Let us start with the case of $p^d_2 > p^d_1$. Using equilibrium conditions for $p^u_{k,m}$ where $k = \{l, h\}$ and $m = \{1, 2\}$ for both one-quality (1Q) and two-quality (2Q) cases, we obtain the following:

$$
\underbrace{\frac{p^u_{l,2}}{\varepsilon^u} - (p^u_{l,2} - c(q_l))}_{2Q} > \underbrace{\frac{p^u_1}{\varepsilon^u} - (p^u_1 - c(q_l))}_{1Q},
$$

where the subscript k is suppressed for the one-quality case. As the above equation decreases $p^u_{k,m}$, we obtain $p^u_{l,2} < p^u_1$. Thus, $p^u_{l,2} < (>) p^u_1$ if and only if $p^d_2 > (<) p^d_1$. □

Proof of Proposition 3. For this proof, we need the equilibrium conditions for both one-quality and two-quality cases, which are reproduced below:

$$
\frac{p^u_1}{\varepsilon^u_1} = \frac{g(q_l)p^d_1}{\varepsilon^d_1} \iff \frac{\frac{p^u_1}{\varepsilon^u_1}}{\frac{g(q_l)p^d_1}{\varepsilon^d_1}} = 1; \tag{40}
$$

$$
\frac{p^u_{l,2}}{\varepsilon^u_{l,2}} + \frac{p^u_{h,2}}{\varepsilon^u_{h,2}}\lambda_h = \frac{p^d_2}{\varepsilon^d_2}\{g(q_l) + [g(q_h) - g(q_l)]\lambda_h\} \iff \frac{\frac{p^u_{l,2}}{\varepsilon^u_{l,2}}}{\frac{g(q_l)p^d_2}{\varepsilon^d_2}} = \frac{\left\{1 + \frac{[g(q_h)-g(q_l)]}{g(q_l)}\lambda_h\right\}}{(1 + \frac{\pi_{h-l}}{\pi_l}\lambda_h)}. \tag{41}
$$

Let us start with the case in which the normalized profit for high-quality buyers is greater on the developer's side, i.e.,

$$
\frac{\pi_{h-l}}{\pi_l}\lambda_h < \frac{[g(q_h) - g(q_l)]}{g(q_l)}\lambda_h \iff 1 + \frac{\pi_{h-l}}{\pi_l}\lambda_h < 1 + \frac{[g(q_h) - g(q_l)]}{g(q_l)}\lambda_h \iff 1 < \frac{1 + \frac{[g(q_h)-g(q_l)]}{g(q_l)}\lambda_h}{1 + \frac{\pi_{h-l}}{\pi_l}\lambda_h}.
$$

Using Equation (40), we obtain the following:

$$\frac{\frac{p_1^u}{\varepsilon_1^u}}{\frac{g(q_l)p_1^d}{\varepsilon_1^d}} < \frac{\left\{1 + \frac{[g(q_h)-g(q_l)]}{g(q_l)}\lambda_h\right\}}{(1 + \frac{\pi_{h-l}}{\pi_l}\lambda_h)}.$$

Using Equation (41), we obtain the following:

$$\frac{\frac{p_1^u}{\varepsilon_1^u}}{\frac{p_1^d}{\varepsilon_1^d}} < \frac{\frac{p_{l,2}^u}{\varepsilon_{l,2}^u}}{\frac{p_2^d}{\varepsilon_2^d}}.$$

By Lemma 1, which states that the price on the developer's side and the buyer's side will move in opposite directions by the introduction of one more quality level, and the above equations, we find that $p_1^d \leq p_2^d$ and $p_1^u \geq p_{l,2}^u$. The opposite case will follow similarly. □

Proof of Corollary 1. From Proposition 3, we know that the price on the developer's side decreases upon the introduction of new quality access if the following condition holds:

$$\frac{\pi_{h-l}}{\pi_l} > \frac{[g(q_h) - g(q_l)]}{g(q_l)}. \tag{42}$$

In the special case of $g(q_k) = 1 \ \forall q_k$, Equation (42) becomes as follows:

$$\frac{\pi_{h-l}}{\pi_l} > 0,$$

which is true in the case in which quality differentiation is profitable. □

Proof of Proposition 4. This proof shows that the platform offers a better price-quality ratio in the case of one-quality offering. This implies the following:

$$\underbrace{\frac{p^u}{q_l}}_{\text{Platform (two-sided)}} \leq \underbrace{\frac{p_{os}^u}{q_l}}_{\text{One-sided}}, \tag{43}$$

where the subscript os indicates one-sided market.

We start by comparing the profit functions for two different cases. For simplicity, we normalize c^d to zero.

$$\begin{aligned} \Pi_{\text{one-sided}} &= [p_{os}^u - c(q_l)]D^u \equiv \Psi(p_{os}^u, q_l). \\ \Pi_{\text{platform}} &= [p^u + p^d - c(q_l)]D^u D^d \equiv \Psi(p^u, q_l)D^d + p^d D^u D^d. \end{aligned} \tag{44}$$

The one-sided monopolist's profit maximization problem is given in Equation (20). Let the optimal solution be $\bar{x} = (\bar{p}^u_{os}, \bar{q}_l)$ for the one-sided monopoly problem and $\widetilde{x} = \left(\widetilde{p}^u, \widetilde{q}_l, \widetilde{p}^d\right)$ for the two-sided platform. As Π_{platform} is maximized at $\widetilde{x} = \left(\widetilde{p}^u, \widetilde{q}_l, \widetilde{p}^d\right)$, the following inequality should hold:

$$\Psi(\widetilde{p}^u, \widetilde{q}_l)\widetilde{D}^d + \widetilde{p}^d \widetilde{D}^u \widetilde{D}^d \geq \Psi(\bar{p}^u_{os}, \bar{q}_l) + \widetilde{p}^d \bar{D}^u \widetilde{D}^d, \tag{45}$$

where $\widetilde{D}^u$ and $\widetilde{D}^d$ indicate the demand for buyers and developers, evaluated at $\widetilde{x}$, respectively. As $(\bar{p}^u_{os}, \bar{q}_l)$ is the optimal solution for $\Pi_{\text{one-sided}}$, we have the following:

$$\Psi(\bar{p}^u_{os}, \bar{q}_l) \geq \Psi(\widetilde{p}^u, \widetilde{q}_l). \tag{46}$$

Equations (45) and (46) imply the following:

$$\Psi(\widetilde{p}^u, \widetilde{q}_l)\widetilde{D}^d + \widetilde{p}^d \widetilde{D}^u \widetilde{D}^d \geq \Psi(\widetilde{p}^u, \widetilde{q}_l) + \widetilde{p}^d \bar{D}^u \widetilde{D}^d. \tag{47}$$

Assuming $\widetilde{p}^d \widetilde{D}^d > 0$, this implies that $\widetilde{D}^u \geq \bar{D}^u$ or $\underbrace{\frac{p^u}{q_l}}_{\text{Platform (two-sided)}} \leq \underbrace{\frac{p^u_{os}}{q_l}}_{\text{One-sided}}$. □

Proof of Corollary 2. It is guaranteed by Proposition 4.